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STRATEGIC SUPPORT TO STUDENTS' COMPETENCY DEVELOPMENT IN THE MATHEMATICAL MODELLING PROCESS: A QUALITATIVE STUDY

ABSTRACT

This article reports on third-year mathematics students' competency and sub-competency development through providing intentional support in the learning of mathematical modelling. Students often experience modelling as difficult, and obstructions in the modelling process can lead to a dead end. Literature reports confirm that the modelling task is central in the modelling experience and a carefully planned task, aligned with a suitable activity sheet, can be used as a scaffold in learning mathematical modelling. Hence, this inquiry was conducted to provide a scaffold, as strategic support, for students' mathematical modelling competency development in the early stages of a modelling cycle. Guided by the framework of the Zone of Proximal Development, key elements suggested by the metaphor scaffolding are considered in the learning experience. Based on an analysis of activity sheets collected through group work, an example of a realistic and an unrealistic solution is presented, and students' development of mathematical modelling competencies is argued. Finally, suggestions for intentional support in the modelling process are discussed.

Keywords: *Activity sheet; mathematics applications; mathematical modelling cycle; mathematical modelling competencies; scaffolding; strategic support.*

1. INTRODUCTION

Niss, Blum and Galbraith (2007) explained that the generic purpose of building and making use of a model is to understand problems in some segment of the real world (e.g., nature, society or culture, everyday life, scientific and scholarly disciplines, and others). But then, for teaching in particular, Ikeda (2013: 255) highlighted two primary pedagogical aims – modelling for teaching can firstly be identified as the content area modelling "through modelling treated as the objective" and secondly, the construction of mathematical knowledge "through modelling treated as a method to achieve a goal".

In this study, the emphasis on learning mathematical modelling is the modelling process, and moving successfully through the different phases of a modelling cycle. Thus, the content area focuses on modelling, and not so much on the construction of new mathematical knowledge. The notion of a mathematical modelling perspective, described in the seminal work in the field of educational mathematical modelling by Niss *et al.* (2007: 9-10), is explained through the initial conceptualisation of some problem situation, and mathematical modelling is then “the entire process consisting of structuring, generating real world facts and data, mathematising, working mathematically and interpreting/validating (perhaps several times round the loop)”.

The modelling process is cyclic in nature and includes a number of phases. Although a variety of cycles exist in the literature, the original modelling cycle from Blum and Leiβ (2007) seems most useful for research purposes in mathematics education. This particular cycle includes individual steps (that separate the phases of the mathematical modelling process): (i) constructing, (ii) simplifying/structuring, (iii) mathematising, (iv) working mathematically, (v) interpreting, (vi) validating and (vii) exposing.

In this inquiry, considering the limited experience of participants in mathematical modelling and the pragmatic lens of the research, a simplified scheme seemed appropriate to support student teachers to solve mathematical modelling tasks e.g., the “solution plan” in Blum and Borromeo Ferri, (2009). Furthermore, the mathematical modelling cycle proposed by Balakrishnan, Yen and Goh (2010: 250) seemed suitable for attempting modelling tasks for the first time. In this cycle, student teachers were introduced to the nature of the mathematical modelling process and could acquaint themselves with the various elements. The cycle comprised four elements (indicated as steps), namely (i) “*mathematisation*” that involves understanding the problem, making assumptions to simplify the problem and representing the problem in mathematical form; (ii) “*working with mathematics*” that includes solving the mathematical problem using mathematical methods and tools; (iii) “*interpretation*” that contains interpreting the mathematical solution within the context of the original problem and (iv) “*reflection*” that comprises reviewing the assumptions and the limitations of the mathematical model and the solution to the problem, reviewing the mathematical methods and tools used, and improving on the mathematical model. This particular modelling cycle (Balakrishnan *et al.*, 2010) has been developed to support secondary school mathematics teachers in implementing mathematical modelling in Singaporean schools.

Despite numerous challenges in the changeover between the different stages of the cycle, Stillman *et al*. (2007) highlight remarkable mathematical accomplishments by students moving through the modelling cycle. Some of these accomplishments include much-needed modelling competencies, described by Niss *et al*. (2007: 12) as “the ability to identify relevant questions, variables, relations, or assumptions in a given real-world situation, to translate this into mathematics, and to interpret and validate the solution of the resulting mathematical problem in relation to the given situation”. In order to avoid a “dead-end” in the mathematical modelling process or a possible lack in mathematical understanding, experts in the field propose that challenges should be addressed through a strategic intervention (Schukajlow, Kolter & Blum, 2015). Schukajlow and colleagues reported on the positive effects of a step-by-step instrument, the “solution plan”, on learning mathematical modelling though a quasi-experimental design study. Grade 9 German students who used the solution plan outperformed the other students in solving modelling problems related to the theorem of Pythagoras. The solution plan provides a modelling schema for students – understanding the task, searching for the mathematics, using the mathematics and explaining the results. Some studies reported on modellers experiencing difficulties in the modelling process (compare Blum & Borromeo Ferri, 2009), while others reported on the positive outcome of intentional support activities (compare Buchholtz, 2017). The latter explained the way in which an "out-of-school" activity (in a European setting), such as a mathematical city walk, could be used to develop competences in mathematising.

This inquiry forms part of a broader research project focusing on a strategy to integrate mathematical modelling in the formal education of mathematics student teachers (Grade 10–12). Early findings (Durandt & Jacobs, 2018) from the broader project revealed that a well-planned set of activities is required for the professional development of mathematics student teachers. As a result, the authors implemented a strategic intervention in the form of an activity that involved two mathematics application tasks. The aim of these tasks was to support students in the development of competencies and the sub-competencies required for the mathematical modelling process, guided by the notions of the Zone of Proximal Development (ZPD) (Vygotsky, 1978), before they are exposed to more challenging tasks. The modelling task is central in the mathematical modelling learning experience and the *Guidelines for Assessment and Instruction in Mathematical Modelling Education* (GAIMME) report (COMAP-SIAM, 2016) explains the transformation from a mathematics word problem to a modelling task as follows (see Figure 1). A traditional mathematics word problem (requiring traditional problem solving) is transformed to a mathematics application problem (requiring mathematical applications) *by adding context and meaning*, and finally to a mathematical modelling problem (requiring the complete modelling process) *by adding interpretation*. With *mathematical applications*, the focus was on the flow "mathematics → reality" and to more generally emphasise the objects involved in the real-world (the parts of the real world that are made accessible to a mathematical treatment and to which corresponding mathematical models exit); while with *mathematical modelling,* the focus was on the direction "reality → mathematics" and the general emphasis was on the process it involved (Niss *et al.,* 2007: 10).

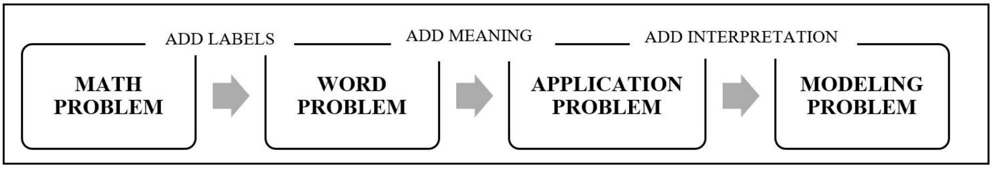

Figure 1: Transforming a mathematical problem into a modelling problem (COMAP-SIAM, 2016: 12)

The two tasks that were used in the intervention can be seen in Figures 2 and 3. Both of these tasks can be classified as mathematical applications (according to GAIMME's explanation) and solving these tasks would merely require "stripping" the words from the problem, and the modelling process would be limited to mathematisation, mathematical procedures, and a direct contextual interpretation. The participants in this inquiry were thus expected to use their mathematical knowledge to solve the tasks. A more open-ended modelling task would require substantial modelling demand from students, but the focus in this inquiry was to support novice modellers (due to their limited exposure to such tasks before) and to avoid possible "dead ends" in the modelling process that could handicap students' competency development. Through the tasks, the authors attempted to follow at least five of the six elements using the metaphor of scaffolding identified by Wood, Bruner and Ross (1976): (i) to enlist students' interest in these mathematical application tasks, (ii) to simplify the tasks by regulating feedback according to particular sub-questions asked, (iii) to keep the students focused on the task by means of well-planned facilitation, (iv) to emphasise correspondences and discrepancies by discussing results in groups, and (v) to discuss a possible solution for a mathematical application problem in groups.

Mathematical Application Task 1

The daily production of a sweet factory consists of at most 100kg chocolate covered nuts and at most 125kg chocolate covered raisins, which are then sold in two different mixtures. **Mixture A** consists of equal amounts of nuts and raisins and is sold at a profit of R5 per kilogram. **Mixture B** consists of one third nuts and two thirds raisins and is sold at a profit of R4 a kilogram. Let there be x kg of mixture A and y kg of mixture B.

1. Express the mathematical constraints that must be adhered to.
2. Write down the objective function that can be used to determine maximum profit.
3. Represent the constraints graphically and clearly show the feasible region.
4. Use the graph and determine the maximum profit obtained.

Figure 2: Mathematical Application Task 1

Mathematical Application Task 2

A nutritionist is performing an experiment on student volunteers. He wishes to feed one of his subjects a daily diet that consists of a combination of three commercial diet foods: MiniCal, LiquiFast and SlimQuick.

For the experiment, it is important that the subject consumes exactly 500mg of potassium, 75g of protein, and 1150 units of vitamin D every day.

The amounts of these nutrients in one ounce of each food are given in the table. How many ounces of each food should the subject eat every day to satisfy the nutrient requirements exactly?

	MiniCal	LiquitFast	SlimQuick
Potassium (mg)	50	75	10
Protein (g)	5	10	3
Vitamin D (units)	90	100	50

Figure 3: Mathematical Application Task 2

The purpose of this inquiry is to report on third year mathematics students' mathematical modelling competency development by providing strategic support in the learning of mathematical modelling as they take part in an activity containing two mathematical application tasks. The two activity questions are: (1) can novice modellers move successfully through the elementary stages of the modelling cycle by exposing them to mathematical application tasks, and (2) have they learnt mathematical modelling competencies and sub-competencies through this learning experience? This inquiry, that forms part of a broader research project, was conducted to provide a scaffold as intentional support for students' competency and sub-competency development in the early phases of a modelling cycle while they are exposed to mathematical application tasks, and before they are exposed to more open-ended mathematical modelling tasks.

2. THEORETICAL PERSPECTIVES

One characteristic of the mathematical modelling activities in this intervention was the team problem-solving approach via collaboration. This collaboration was enhanced by the two underlying notions of the ZPD (Vygotsky, 1978) and scaffolding. The most widely known definition of ZPD is "the distance between the actual developmental level, as determined by independent problem solving, and the level of potential development, as determined through problem solving under adult guidance or in collaboration with more capable peers" (Vygotsky, 1978: 33). Following on the Western interpretation of Vygotsky's work, the notion of *scaffolding* developed almost 40 years ago (Wood *et al.,* 1976). Other researchers in mathematics education (Anghileri, 2006; Schukajlow, *et al.,* 2015) have also acknowledged this metaphor, which refers to the intended systematic support for an individual student by a knowledgeable adult. In this inquiry, a collaborative ZPD was made possible by student interaction in small groups as they participated in mathematical modelling activities. This provided a support structure or "scaffold" for students' competency development and they could learn from one other's knowledge.

Furthermore, the notion of the *Zone of Proximal Development* can be seen as the underlying theoretical lens to explain students' cognitive development in mathematical modelling. Student cognitive development was potentially promoted by following the steps in the mathematical modelling cycle and by collaborating in groups when involved in a learning activity containing two mathematical application tasks – both serving as a scaffold to support mathematics students' increasing independence as their understanding of mathematical modelling became more secure. Wood *et al.* (1976), also cited by Anghileri (2006:34), identified six key elements using the metaphor of scaffolding, namely:

i. *recruitment* – to enlist the students' interest in a mathematical modelling activity, to willingly adhere to the requirements of the activity;

ii. *reduction in degrees of freedom* – to simplify mathematical modelling tasks by regulating feedback according to the particular phases of modelling cycles;

iii. *direction maintenance* – to keep the students focused on the task by means of well-planned facilitation by the facilitator;

iv. *marking critical features* – to emphasise correspondences and discrepancies by discussing results;

v. *frustration control* – to respond to students' reactions when participating in mathematical modelling tasks; and

vi. *demonstration* – to discuss a possible solution for a real-life contextual problem.

All of these key elements were considered in planning and implementing the structured intervention in this inquiry, but in particular the first four as well as the last element.

3. RESEARCH APPROACH AND METHOD

The participants, third-year mathematics student teachers at a public Johannesburg University, characterised by a variety of demographical profile elements, are identified in Table 1. The research was conducted from a pragmatic viewpoint (Creswell, 2013), by collecting qualitative data, and participants were arranged into 10 groups according to ability. Each group had to contain at least one high achiever (an average mathematics mark of equal or above 70%), one moderate achiever (an average mathematics mark of between 50% and 70%), and one low achiever (an average mathematics mark of below 50%). The participants were all enrolled in a third-year mathematics course for teachers and their average mathematics course marks were used for the group allocation. The rationale for the group selection of high, moderate and low achievers was due to the complexity of real-world contexts, the high cognitive demand of mathematical modelling tasks (although in this inquiry participants were exposed to a mathematical application task and not yet open-ended modelling task), and student teachers' lack of experience in mathematical modelling. Small group sizes were practical and the idea was to promote participation of all members. Stratified sampling procedures were used, which are also convenient (due to respondents' availability) and single stage, as the researcher had access to the names of all student teachers in the group and could sample directly (Creswell, 2003).

Table 1. Demographical profile elements of participants

Profile variable (N=55)		**N**	**%**
Gender (N=43)	Female	17	39.5
	Male	26	60.5
Ethnic Group (N=42)	Asian, Indian, Coloured	2	4.8
	Black	32	76.2
	Coloured	3	7.1
	White	3	7.1
	No response	2	4.8
Home Language (N=43)	Afrikaans	4	9.3
	English	5	11.6
	Indigenous	33	76.7
	No response	1	2.3

Participants were exposed to the mathematical modelling activity (see Figures 2 & 3) that contained two contextual problem-solving examples that required little ambiguity from groups about their strategies and methods, and an activity sheet (similar to the solution plan from Schukajlow *et al*., 2015) containing the fundamental steps in solving modelling tasks (understanding the task, searching for mathematics, using mathematics and explaining results). The information in both selected examples had already been carefully defined, containing all the necessary data to formulate a model, and called for a specific procedure to be employed. Thus, the authors expected participants to move successfully through the modelling cycle. The activity, involving both tasks, was conducted during a 110-minute official class timeslot. During the activity, the researchers were present to observe group discussions. Apart from the theoretical basis that informed this inquiry, the rationale for exposing student teachers to mathematics application tasks was informed by Chan's viewpoint (2013) that a possible starting point for the integration of mathematical modelling into the classroom can be identified via simple tasks. The researchers intended to design an activity, not only to support students in the development of mathematical competencies in the modelling process, but also to enhance collaboration between group members, and to stimulate favourable participant attitudes (also compare Durandt & Jacobs, 2018).

Task 1, on the daily production of a sweet factory (see Figure 2), required students in groups to express mathematical constraints and to represent the given data, then graphically represent the constraints, and to make use of the graph to determine maximum profit (Bester *et al*., 1998: 113). Task 2 (see Figure 3), adapted from Stewart, Redlin and Watson (2012), provided information on the potassium, protein and vitamin D content in one ounce of commercial diet food. Information was presented in the form of a table in order to compare the three different brands of diet food. Groups were expected to calculate how many ounces of each type of food should be taken per day to satisfy daily nutrient requirements. Groups submitted their strategies and mathematical solutions on a pre-designed activity sheet, designed according to the elementary stages of a modelling cycle to provide students with a scaffold.

4. ANALYSIS AND FINDINGS

Qualitative data were analysed by following the direct content analysis method (Hsieh & Shannon, 2005). This method of analysis uses existing research to guide the variables of interest (proposed categories) and to devise operational definitions for these variables. Hsieh and Shannon (2005) argued that the use of existing research could not only provide guidance on the variables of interest, but also assists in focusing the research questions. Two proposed categories originated from the theory on students' mathematical modelling competencies (Stillman, *et al.,* 2007) on the one hand, and the principles (accurate, realistic, precise, practical and robust) to evaluate a mathematical model (Meyer, 2012) on the other hand. The two coding categories, *mathematical modelling competence* (with a differentiation between modelling sub-competence and mathematical sub-competence) and *model capability*, guided the analysis of the activity sheets by the different groups that were collected during the intervention (see Table 2). The directed content analysis method also suited the researchers' pragmatic approach, and existing theory on mathematical modelling was supported and extended by means of this method.

The first category, *mathematical modelling competence*, describes the ability of participants to identify relevant questions, variables, relations, or assumptions in a given real-world situation. Examples in this paper include the modelling task described earlier that required a mixture of a certain food substance according to certain conditions and to translate this into mathematics and to interpret and validate the solution of the resulting mathematical problem in relation to the given situation (compare Niss *et al*. 2007). Within this first category, modelling sub-competence is underscored as a sub-category focusing particularly on the modelling cycle, real-world meaning and real-world solution - or how participants (in groups) followed the steps of the modelling cycle, progressed from meaning towards a possible solution, and then interpreted the solution within the given real-world context created through the mathematical application tasks that were applied. Also embedded in this first category is mathematical sub-competence as a sub-category, focusing on participants' mathematical abilities – or if participants (in groups) used relevant and accurate mathematical representations of the real-world problem in their selection of applicable mathematical content knowledge, and if they worked correctly with the mathematics.

The second category, *model capability*, describes if participants (in groups) considered the evaluation criteria of the mathematical models produced by the various groups. This relates to their particular model's precision, validity and applicability and is described by the model's accuracy (if the model output values are correct or near to correct), realism (if the model is based on original correct assumptions), precision (if the model output values is definite mathematical quantities or if they fall in a specific interval), practicality (if the model output values provide a sensible solution) and robustness (if the output values are protected against errors in the input data). Owing to the nature of the mathematical application tasks used in this inquiry, which contained clearly defined data, it was expected that the different group models would mostly satisfy Meyer's (2012) criteria.

Table 2 shows an analysis of participant groups' activity sheets - how the different groups displayed mathematical modelling competence and model capability by solving the two mathematical application tasks (compare Figures 2 and 3).

5. DISCUSSION

The mathematics application activity in this inquiry involved two different tasks and did not require the participant groups to progress through the complete modelling cycle. It also did not require them to consider the implications of their decisions, or to discuss the validity and applicability of their selected mathematical models. The focus of the two tasks was on the development of mathematical and modelling competence, and the researchers attempted to use the modelling cycle as a metacognitive tool to expose students to the early stages of the modelling cycle.

In Figure 4, the mathematical and modelling competence of group 1 is illustrated. The Figure indicates the manner in which the group structured the relevant information in a table, how they expressed the mathematical constraints as two inequalities having written down an objective function to determine maximum profit, and finally how they presented the problem graphically. In addition, the objective function (as the dotted line) and final solution (at point A) are depicted in the figure.

All 10 groups showed *mathematical modelling competence*, particularly modelling sub-competence in their documents, as they proceeded successfully through the first two fundamental phases of the modelling cycle (Balakrishnan *et al*., 2010). All groups identified relevant information and made simplified assumptions about the real-world contexts in both examples included in the task. Furthermore, all groups showed mathematical sub-competence as findings. They indicated the mathematical representation used, as well as the content selection and mathematical operations performed by the different groups. All 10 groups introduced relevant variables and represented the application problems mathematically (given in two examples), although only eight of these representations were accurate. The two inaccurate representations, from group 8 and 9, contained minor mathematical errors. The entire class selected appropriate mathematical formula to solve the problems, and seven groups investigated more than two mathematical procedures, such as solving equations simultaneously, determining solutions graphically, and performing matrix operations. Seven of the groups applied the selected mathematical content knowledge correctly, but the documents from three of the groups indicated mathematical errors in procedures.

Table 2. Analysis of group activity sheets

Initial Coding Categories	**Groups**										
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**Total**
Category: Mathematical Modelling Competence											
Modelling Sub-Competence											
Moving successfully through the first two stages of the modelling cycle.	X	X	X	X	X	X	X	X	X	X	10
Identifying, from the available information, what is relevant and what is irrelevant.	X	X	X	X	X	X	X	X	X	X	10
Making simplified and relevant assumptions about the situation to enable mathematics to be applied.	X	X	X	X	X	X	X	X	X	X	10
Mathematical Sub-Competence											
Recognising relevant variables.	X	X	X	X	X	X	X	X	X	X	10
Accurately representing the real-world problem mathematically.	X	X	X	X	X	X	X	-	-	X	8
Inaccurately representing the real-world problem mathematically.	-	-	-	-	-	-	-	X	X	-	2
Selecting appropriate mathematical formula – consistent with the representation.	X	X	X	X	X	X	X	-	-	X	8
Selecting appropriate mathematical formula – inconsistent with the representation.	-	-	-	-	-	-	-	X	X	-	2
Applying mathematical content knowledge correctly.	X	X	-	X	X	X	-	-	X	X	7
Applying mathematical content knowledge incorrectly.	-	-	X	-	-	-	X	X	-	-	3
Investigating more than two mathematical procedures to solve the problem.	X	X	-	-	X	X	X	X	X	-	7
Category: Model Capability											
Accurate (model output expected to be near to correct)	X	X	-	X	X	X	-	-	X	X	7
Realistic (model based on original correct assumptions)	X	X	X	X	X	X	X	X	X	X	10
Precise (model output values should be definite mathematical quantities or falling in a closed interval)	X	X	-	X	X	X	X	X	X	X	9
Practical (sensible solutions provided)	X	X	-	X	X	X	-	-	X	X	7
Robust (relatively safe from errors)	X	X	X	X	X	X	X	X	X	X	10

Owing to the carefully defined mathematical application tasks included in this inquiry, little ambiguity about the model and procedures was expected from the groups. As anticipated, all groups formulated realistic models based on correct assumptions and prior exposure to relevant mathematical content knowledge. Groups displayed *model capability* by means of its accuracy, precision and practicality (Meyer, 2012). These criteria depended on the mathematical competencies of groups; hence, seven of the groups formulated output values expected to be correct (referring to accuracy). The same seven groups provided sensible solutions (directed towards practicality), while the models from nine of the groups generated definite output values (indicating precision). In the case of group 8, who struggled to represent the real-world problem mathematically, definite output values were calculated, given by , but the values were impractical and inaccurate. Noticeably, a negative number of ounces of LiquiFast () is an unrealistic solution. The researchers expected that the particular group would realise the negative value is impossible and comment on that or check their mathematical calculations.

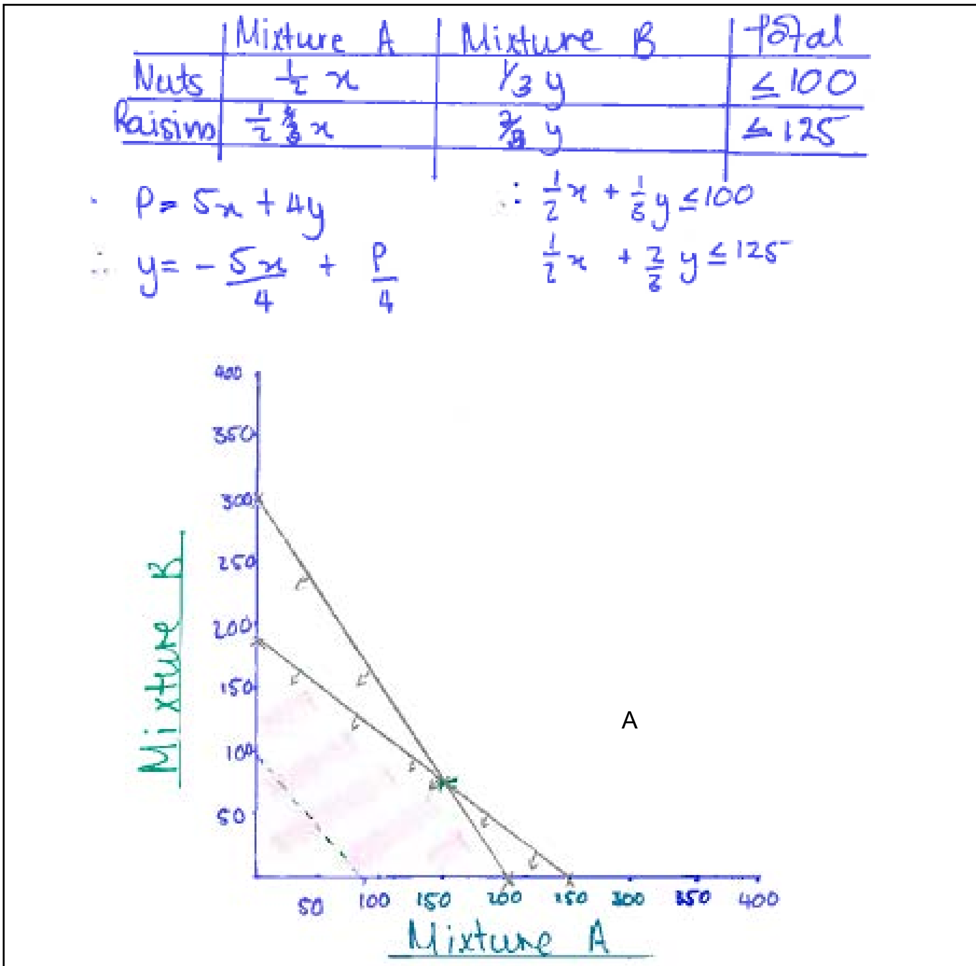

Figure 4: Example from group 1 illustrating mathematical and modelling competencies

6. CONCLUSION

This inquiry reports on third-year mathematics students' mathematical modelling competency (and sub-competency) development through intended strategic support in the learning of mathematical modelling. The researchers attempted to use the modelling cycle as a metacognitive tool and to use the mathematics application tasks, together with an activity sheet linking the key steps in the modelling cycle as a scaffold in the modelling process. The rationale for exposing student teachers to mathematics application tasks was informed by Chan's viewpoint (2013) that a possible starting point for the integration of mathematical modelling into the classroom could be identified via simple tasks. Furthermore, the different steps in the modelling cycle could be a potential cognitive barrier for participants. Thus, the aim of these tasks was to support students in the development of competencies and sub-competencies required for the mathematical modelling process, guided by the notions of the Zone of Proximal Development (ZPD) (Vygotsky, 1978), before they are exposed to more open-ended and more challenging modelling tasks. Knowing the modelling task is central in the mathematical modelling learning experience, and that modellers often experience blockages in the modelling process, well-planned and structured support was found to be necessary in the case of novice modellers. The structured support included a solution plan containing key steps that are linked to the fundamental phases of the modelling cycle – understanding the task, searching for the mathematics, using the mathematics, and explaining the results.

In this inquiry, findings confirmed that inexperienced modellers could move successfully through the elementary stages of the modelling cycle (Balakrishnan *et al.,* 2010) by exposing them to an easier, clearly defined mathematical modelling activity including two mathematical application tasks. The tasks required mixing particular food substances according to certain conditions. Data were analysed via the direct content analysis method and existing research categories guided the coding categories. Most participant groups displayed mathematical and modelling sub-competencies and they showed an understanding of mathematical models (within a clearly defined situation). Some groups displayed realistic answers, while others had unrealistic answers. It was expected that the groups with unrealistic answers would attempt some effort to unpack the problem or move through the modelling cycle for a second time, in searching for an explanation for their particular solution, but they did not.

In conclusion, the participants revealed themselves as mathematical modellers, although the modelling process was limited to mathematisation, mathematical procedures, and a direct contextual interpretation. This inquiry provides a basis for further strategic support and continued exposure to more open-ended modelling tasks.

7. ACKNOWLEDGEMENT

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