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The concrete-representational-abstract sequence of instruction in mathematics classrooms

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The purpose of this paper is to explore how master mathematics teachers use the concrete-representational-abstract (CRA) sequence of instruction in mathematics classrooms. Data was collected from a convenience sample of six master teachers by observations, video recordings of their teaching, and semi-structured interviews. Data collection also included focus-group interviews with learners. In South Africa, master teachers are considered expert teachers in their discipline. The master teachers in this study were selected by the Department of Education based on their many years of teaching experience; in addition, these selected master teachers taught at six different Dinaledi schools in KwaZulu-Natal (South Africa). A key finding of this research demonstrated that the use of the CRA instructional sequence was paramount for the effective teaching of mathematics. This instructional sequence was found to be predetermined as well as intuitive. The CRA instruction may be used in classrooms where learners are not streamed into ability levels, as is the case with the majority of schools in South Africa. These findings are important for shaping both teacher and curriculum development.

Keywords: Visualisation, representations, activity theory, CRA sequence

Introduction

Success in mathematics is becoming increasingly important for learners since it is essential for a variety of employment opportunities (Witzel, Riccomini & Schneider, 2008). Our many years of experience in the field of mathematics education have convinced us that the teaching of mathematics can be highly frustrating. For example, algebra often requires the manipulation of equations that have little to do with the

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purpose of the equation; the arbitrary symbols may be manipulated with little regard to the numerals in the original equation (Witzel, 2005). Researchers have shown that the use of the CRA sequence of instruction has been very effective and beneficial to learners who struggle with understanding mathematical concepts and procedures (Flores, 2009; Witzel *et al.*, 2008). Anecdotal evidence and research suggest that learners often claim that mathematics is difficult and abstract. Some teachers claim that learners are no longer prepared to work hard. Furthermore, these teachers maintain that either they do not have the proper resources to teach mathematics adequately or they find learners uncooperative and unprepared. Despite these negative feelings, there are teachers who are able to assist their learners in making meaning of what may sometimes be considered abstract concepts in mathematics. This, inevitably, leads to learners finding the mathematics being taught interesting and being able, to some extent, to understand abstract mathematical concepts.

It was with this in mind that we explored the approach used by master teachers in their mathematics classrooms. Master teachers are generally knowledgeable, competent teachers who have a record of excellent achievement in the Senior Certificate Examinations (HEDCOM Secretariat, 2006). These teachers are highly performing successful teachers who have the skill and knowledge to teach mathematics and can share experience and expertise with other teachers (DoE, 2006). In 2006, the Department of Education (KZN) announced that in terms of teacher development, 120 master teachers would be appointed and an additional 2.400 master teachers would be trained (Makapela, 2007). This announcement was made due to the realisation that many schools in South Africa lacked qualified mathematics and science teachers. In addition, all the master teachers in this study taught at Dinaledi schools. These schools were selected by the National Department of Education with the intention of increasing the participation and performance of Black learners and female learners in mathematics and science. The Dinaledi project was intended as a short-term project providing teaching and learning resources to a limited number of schools. Selected schools were regarded as 'Star' schools in South Africa.

Teaching of mathematics using CRA instruction sequence

Teaching learners mathematics, using concrete objects, pictorial representations, followed by abstract symbols and numerals, is called the Concrete to Representational to Abstract (CRA) instructional strategy (Witzel, 2005). This approach has been found to be useful in increasing the understanding of abstract mathematical concepts and ideas (Witzel, Mercer & Miller, 2003). The CRA instructional approach is a three-stage process. The first stage requires that learners manipulate concrete objects, followed by the pictorial representation of whatever they have done at this stage. The concrete manipulative may be represented by a drawing, picture or diagram. Thus, the teacher uses the same model and procedures to pictorially represent the mathematical process. Thereafter, the problem may be solved using abstract symbolic notation, which involves memorisation of mathematical procedures and continues until the learner learns the procedure or concept automatically (Flores, 2009; Witzel, 2005; Witzel et al., 2008).

When using the CRA method, the sequencing of activities is important. Teachers ought to start at the concrete level before moving to the representational level and, finally, the abstract level. The third part of the sequence, abstract thinking, will only be required if the information cannot be readily represented at a concrete or representational level. To achieve this outcome, teachers need to plan carefully and use innovative strategies in class. The concrete understanding ought to be attempted first by using appropriate concrete objects. Secondly, representational understanding is achieved by using an appropriate drawing technique and, finally, appropriate strategies are used to assist learners in moving towards the abstract level of understanding of the concepts and symbols for a particular mathematical idea using explicit teaching. Teachers should use appropriate concrete manipulatives first. It is also important to ensure that learners acquire, retain and master the mathematics skills at each stage of the instructional sequence (Witzel et al., 2008).

In this study, the master teachers used concrete visual tools when they introduced new topics in mathematics and explained known mathematics concepts. It appeared that these visuals serve as suitable scaffolding in their mathematics lessons. These tools acted as instruments necessary for bridging the space that existed between the ‘real world’ and the ‘abstract world’ of symbolism. In our opinion, visualisation can be précised to imply the ability to form and negotiate a mental image necessary for problem-solving in mathematics. Whilst mathematics encompasses many abstract notions that are difficult to acquire and apply (Kaminski, Sloutsky & Heckle, 2008), visualisation enables abstract notions to be made more accessible to the learner.

Moreover, interactions with concrete objects increase the likelihood that learners would remember stepwise procedures in mathematics, because this allows learners to retrieve information in a variety of ways including visual, auditory, tactile and kinesthetic (Witzel, 2005; Witzel *et al.*, 2008). The idea of using visual skills in a classroom is also supported by Gardner’s theory of multiple intelligences. Despite the value of the multiple intelligences theory and even though visual education is important for learners to successfully interact with shapes (Freudenthal, 1971), visual education is generally an area that is ignored in mathematics classrooms (Hershkowitz, Parzysz & Van Dor-Molen, 1996).

Theoretical framework

Activity theory was used as a framework for this study. This theory is based on the assumption that all human actions are mediated by tools and cannot be separated from the social milieu in which action is carried out. In this study, activity theory provides the framework for describing the structure, development and context for the activities that were supported through the use of visuals as tools. These tools differed based on the context within which each school was located. For example, in teaching transformation geometry, one master teacher used the smart board; another master teacher used a stick with different coloured elastic bands, and another used paper-folding and gestures. These diverse tools were used to teach the concept of rotation and reflection in transformation geometry.

In this study, the activity system under the microscope is the act of teaching in mathematics classrooms. The community with the activity system refers to a group of individuals who share a common objective. Barab, Schatz and Scheckler (2004) proposed that activity theory emphasised the mutual nature of learning and doing, of tool use and community, and of content and context. To clarify, as the learning community within each activity system in the study works, plays, and solves problems together, they develop a new set of values and notions. I use Engeström's (1987, 1993, 1994, 2001) second-generation activity theory model in this study.

To situate the activity theory model within the context of this study, I define the subject as the master teacher; the instruments are the visual tools that are used to teach mathematics, and the object is the development of the mathematics content. The communities in this study are the learners within the mathematics classroom, the master teachers, the staff at each school and the parents within the community. The subject belongs to a community that is governed and mediated by rules and division of labour. Essentially, members of the community collaborate with each other to achieve the outcome of the activity system. The following model (Figure 1) emerged from this study. This model graphically demonstrates how activity theory was used in the teaching and learning of mathematics. The smaller external activity system that is illustrated in the Figure varies for each master teacher.

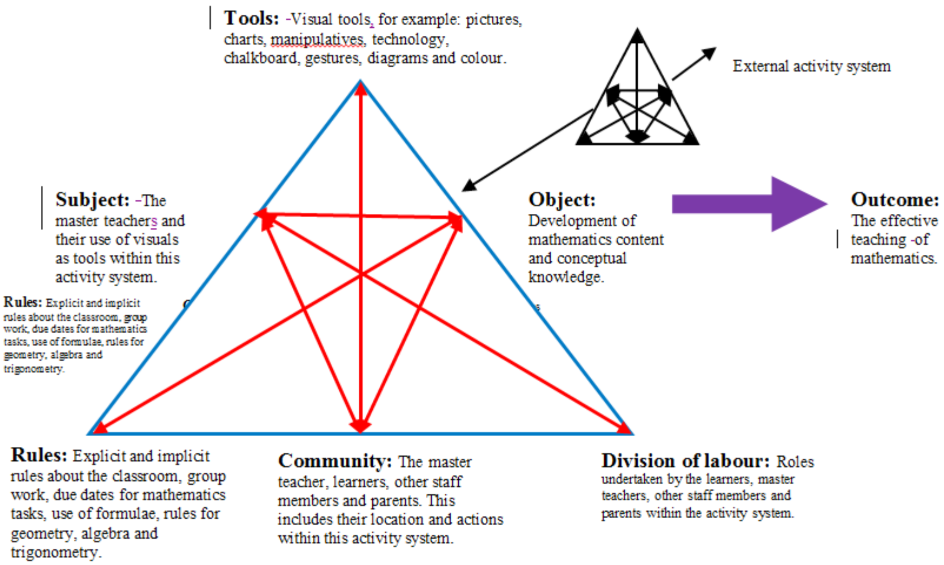

Figure 1: Conceptual model of the human activity system within this study

Adapted from Engeström (1987: 78)

Methodology

Issues of ethics

Ethical clearance was obtained from the research office of the participating university. In addition, each participant was invited in writing to take part in the study and was provided with an informed consent form to peruse at their leisure. Moreover, participants were informed of their right to withdraw from the study without prejudice. Each participant was also informed that s/he would be invited to a dissemination-of-results discussion session at the end of the study.

Participants

The master teachers in this study were generally knowledgeable, good, competent teachers who have a record of excellent achievement in the Senior Certificate Examinations. These teachers are high-performing, successful teachers who have the skill and knowledge to teach mathematics and who can share experience and expertise with other teachers (DoE, 2006). In South Africa, master teachers serve the same purpose as a mentor or expert teacher. They are senior teachers with the potential to mentor new teachers (Naidoo, 2012).

Data collection and analysis

A qualitative interpretation of master teachers' use of visuals as tools within mathematics classrooms was conducted. The qualitative data-collection phase involved five stages of research.

Stage 1: The pilot study

Prior to pilot-testing the questionnaire, items on the questionnaire were discussed with colleagues within similar research areas. After minor edits, the questionnaire was pilot-tested with ten schools. The pilot study assisted in conducting a mock data-collection process in order to ensure the validity and reliability of the research instruments and research methods. The research instruments were subsequently refined based on data collected in the pilot study.

Stage 2: Master teacher questionnaire

In stage 2, a questionnaire was hand-delivered to each of the remaining ten schools. This was a pen-and-paper questionnaire whereby the teachers were asked various questions relating to their teaching strategies and resources used. The questionnaire was analysed and coded in preparation for the next stage of data collection. The final sample was selected for stage 3 of the data-collection process.

Stage 3: Observations and video recordings

Stage 3 involved the observation and video recording of at least three Grade 11 mathematics lessons that were taught by each of the six master teachers. In addition, an observation schedule and researcher field notes were used to record the data collected in this stage of the study. Based on the observations and video recordings, it was evident that the master teachers resorted to using the CRA approach to make the mathematics being taught easier to understand and to make their lessons more interesting and fun. After analysing and coding the observations, Stage 4 commenced.

Stage 4: Master teacher interviews

Each of the six master teachers selected were interviewed using a semi-structured interview schedule. Each interview was recorded. Selections of video clips were provided for the master teachers to view. These video clips focused on the teacher's use of the CRA approach in the classroom. The teachers were asked questions pertaining to the use of this approach in the classroom. Their responses were probed where necessary to ensure that there were no misinterpretations or misunderstandings. After analysing the master teacher interviews, stage 5 was initiated. In this stage of data collection, focus-group interviews (FGI) were held with learners from each of the six schools.

Stage 5: Focus-group interviews with learners

A semi-structured focus-group interview schedule was used and each focus-group interview was recorded. Selected learners were afforded the opportunity to view the same video clips that were shown to their teachers. These selected learners were asked questions about the use of the CRA approach in their classrooms.

Whilst different visuals were used and each master teacher provided different reasons for using visuals, this article focuses on how the master teachers used visuals as *tools* in the mathematics classroom. Based on lesson observations, the scrutiny of video recordings, the master teacher interviews and the focus-group interviews, it was found that, rather than using traditional strategies to teach mathematics, all the master teachers incorporated scaffolding techniques to support their learners' development in mathematics. One of the scaffolding techniques was the master teachers using the CRA approach in their classrooms.

Results

In this study, it was evident that the master teachers deliberately used visuals as tools, while reflecting in action. They used gestures, graphs, geometric shapes, underlining for emphasis and diagrams instinctively while teaching. Apart from the use of gestures, this study also captured the master teachers' use of technology as a visual strategy. Tools such as the smart board and the calculator enabled teachers and learners to display ideas visually and, in this way, facilitate multiple interpretations. To some extent, these visual strategies contributed to meaningful learning of mathematics. For example, if learners were deemed to be experiencing problems with understanding concepts during their teaching, they instinctively resorted to the CRA approach. They used concrete manipulatives, for example a piece of paper, a stick, rubber bands, chairs, bricks and a calculator, in order to enhance the understanding of the mathematical concepts being taught.

The master teacher then switched to the representation level of teaching. In this instance, learners were exposed, where possible, to visual representations of the concept being taught in the form of pictures, diagrams, colour or gestures. By designing lessons based on the CRA sequence of instruction, the teacher encouraged learners to take personal ownership of abstract concepts. Learners made personalised, concrete, meaningful connections by using these appropriate objects (Witzel et al., 2008). Unwittingly, the teachers were engaging the learners in a process of constructing their own meanings in order to deepen understanding.

The learners were afforded an opportunity to practise problem-solving by visually representing their solutions in the form of diagrams or sketches. This led to a continuous teacher-learner interaction until the teacher was satisfied that the learners were able to model the key mathematics concepts at the representational level. When this was achieved, the master teachers proceeded to the abstract level. In this instance, only numbers, mathematics rules and symbols were used. In each master teacher's classroom, learners were afforded many opportunities to practise and demonstrate their mastery of the concepts being taught, before the master teachers moved to a new concept. The master teacher moved among the learners and spoke to them during the lesson. Each learning space became a hive of discussion and activity, and it seemed that discussion between learners was encouraged. What was essential for effective teaching in each of these lessons was the teacher's development of a concept that was structured and went from the concrete to the representational and, finally, to the abstract level. Thus teachers using an algorithmic approach, devoid of visuals, may find it difficult to create an engaging learning environment. The concrete and representational visual tools that were used shaped the way in which the learners built mathematics relationships, in order to advance the comprehension of the abstract mathematical concepts.

Niess (2005) claims that a real challenge inhibiting good teaching is the ability to identify useful problem-solving tools that individual learners would require in the mathematics class. Moreover, the practice of teaching mathematics requires teachers to be both knowledgeable in mathematics and proficient in problem-solving (Kazima, Pillay & Adler, 2008). Anecdotal evidence indicates that mathematics teachers often use visual tools with the intention of assisting learners to grasp a concept in order to improve mathematical conceptual knowledge. In this study, conceptual knowledge is defined as the ability to use previously learnt knowledge in order to solve different problems (Witzel, 2005). Teachers' tacit knowledge and beliefs concerning the teaching and learning of mathematics influenced the teaching and learning of mathematics in their classrooms (Naidoo, 2012).

The master teachers in this study used visuals and concrete tools as a means of scaffolding mathematics thinking and learning. However, before using the concrete and pictorial representations, the teachers had to ensure that their learners were well scaffolded (given guidance and support) (Witzel *et al.*, 2008). Scaffolding provided an alternative method to the traditional 'chalk and talk' approach to teaching. The master teachers used different levels of scaffolding to ensure success in the classroom.

For example, at Rose Secondary while Karyn was revisiting the idea about gradients of lines and the direction in which the lines faced, she used diagrams and gestures. Since the drawing of straight lines using the gradient method is important within analytical geometry, Karyn proceeded to write symbols on the board and drew lines through them as depicted in Figure 1:

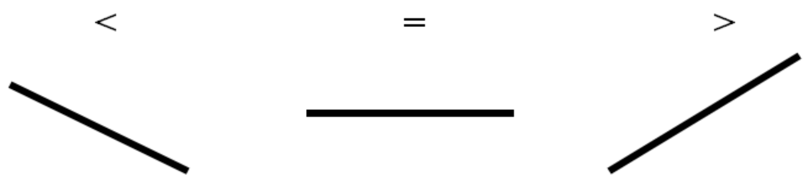

Figure 1: Karyn's use of symbols to represent the slope of a line

In essence, she used this visual (Figure 4) as a means of making the concept of gradients and lines easier to remember. Her discussion revolved around how symbols denoting the greater than, less than or equal to signs facilitated the learners' understanding or knowledge with respect to the slope of the line. When examining the first symbol, it is discernible that the direction of the bottom part of the 'less than' sign is similar to the downward slope of a line. In the second symbol, the bottom part of the 'equal to' sign is similar to a horizontal line. With the last symbol, the bottom part of the 'greater than' sign looks identical to the upward slope of a line. This may seem to be related to the instrumental way of understanding the idea, and is not dissimilar to learning using a pneumonic or even memorising. However, it can be argued that at the most basic level, the teacher was attempting to create an association between the symbolic (<, = or >) and the concrete (the actual line).

When asked about her reasons for using this approach to teach the direction of the slope of lines, Karyn stated that using this visual manner of teaching assisted in making gradients and directions of slopes easier to remember. Karyn believed that the act of drawing a diagram contributed to her learners' comprehension of the mathematical idea or concept. This notion is corroborated by Mudaly (2010) who asserts that, by using diagrams when solving problems, learners can become actively engaged in meaning making. He further asserts that the interpreting of mathematical symbols and words and the act of creating diagrams contribute towards understanding the problem and hence can lead to correct solution of problems. Similarly, if immediate association is established between the sign '>' and the slope of the line, the construction of the direction of the line becomes easier. The following statement by teacher Karyn captures this notion.

> *"... it makes it easier for them (the learners) to remember, can you see every time when I draw a line I use the bottom line and when it is like that I use the other side and when it is horizontal I use the straight line. It's just a way for the learners to remember ..."*

In this instance, remembering is not used in the context of rote memorisation. Karyn supplemented her visuals by using iconic gestures, as shown in Figure 2. Iconic gestures are gestures that resemble their concrete representation (Edwards, 2009). These gestures depict the direction of the slope of each line.

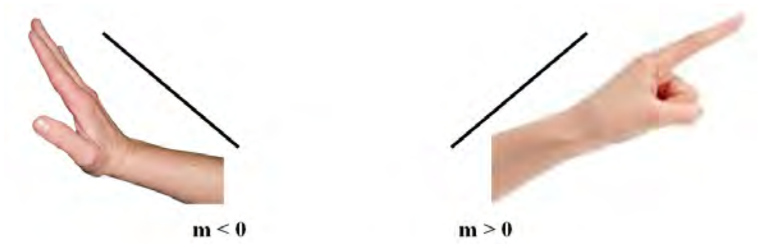

Figure 2: Karyn's hand gestures used to represent the slope of lines

The use of visuals during teacher-learner communication effectively promotes knowledge gain (O'Donnell & Dansereau, 2000). When Karyn used her symbols and her gestures, this seemed to have improved her learners' level of understanding of the concept. Learners who are high achievers usually use a more varied number of strategies to make meaning of mathematics than other students. This is similar to Harries, Barrington and Hamilton's (2006) argument. By using visuals and language, Karyn supported the teaching and learning in her classroom. Her visuals served as a scaffold in the classroom.

Master teachers also used their teaching experience (to some extent, their pedagogical content knowledge), relevant real-life contexts and situations to enable learners to access problems involving abstract concepts more easily.

Sam used his many years of experience as a teacher in order to conduct lessons in his under-resourced and former disadvantaged school, without the luxury of an interactive smart board, an overhead projector, white board or expensive manipulative materials. Sam made effective use of the chalkboard, the classroom environment and his own body to deliver lessons that encouraged meaning making.

Based on the lesson observations and the interviews with Sam as well as those with his learners (during the focus-group interview), it was evident that Sam used visuals in his classroom as a scaffold for the teaching and learning of mathematics. Sam realised that mathematics had to make sense to the learners in order for them to succeed. Sam used visuals that were easily accessible. For example, he used bricks on the walls of the classroom as concrete scaffolding tools. Sam acknowledged that algebra is the gateway to abstract thought and knew that his learners needed to attain success in algebra (Witzel, Mercer & Miller, 2003). Whilst he was teaching number patterns, in order to make the abstract concept of number patterns more concrete and accessible, he spoke about the bricks on the classroom walls. Sam stated that

> *"... I start from the concrete, something they can see ... they can see the pattern in bricks right in front of them ... I think it makes things easier if you are talking about something you can see ...".*

Once this concept of patterns was concretised, Sam proceeded to the representational level of drawing patterns on the board. Finally, he moved on to the abstract level by developing a rule to assist with solving the number pattern.

Sam also used diagrams as a scaffold for mathematical thinking. For example, when he taught an introductory lesson in trigonometry, he used diagrams. By the end of the lesson, learners were expected to calculate trigonometric ratios and identities. He drew different diagrams to serve as support mechanisms for the lesson. The first diagram was used to remind learners of prior knowledge, which would be required to solve problems in the current lesson. Sam focused on using a diagram with which his learners were familiar. This diagram focused on all aspects of the circle. His reason for using the diagram (see Figure 3) is revealed in his response:

> *"... they (the learners) must know when you are talking about a circle ... they don't understand where we are coming with the radius ... it's easy now when I am drawing the radius, they can see now, they can compare ...".*

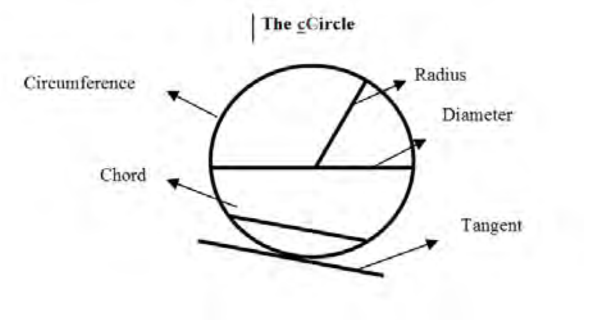

Figure 3: Sam's diagram representing key features of a circle

The diagram became an effective tool to eradicate misconceptions as opposed to an attempt to verbally explain to learners. Furthermore, it was easier to point out on a diagram the differences between, for example, the chord, diameter and the radius, than it is to simply speak about the differences. Thus, the use of the diagram offered the learners a more integrated conceptual understanding of trigonometric ratios and identities. By creating this easy to see, relevant and meaningful diagram, the teacher may progress to Level 3 scaffolding.

Level 3 scaffolding refers to the use of tools with the aim of developing conceptual understanding. Each master teacher used concrete materials and mathematics manipulatives with the intention of making the abstract concepts in mathematics more accessible. Scaffolds that were used became tools and strategies, which assisted learners in achieving a deeper level of conceptual understanding. Scaffolding encouraged divergent and creative thinking in the mathematics classroom. The master teachers were able to assist more learners by using their visuals in this manner.

For example, in Maggie's classroom, it was apparent that Maggie used the smart board not only for demonstration, but also as a stimulus for discussion (Ainley, 2001). Maggie attempted to draw on her learners' prior understanding of a particular mathematics concept. She was teaching her learners that different triangles having the same base and lying between the same two parallel lines would have equal areas. She believed that if she merely wrote the statement her learners would not understand, it would be at an abstract level of thinking. She was of the opinion that it was necessary to teach this at a representational level with diagrams for learners to grasp the concept. This representation was followed by an interactive discussion about the triangles and about the area of the shapes. This was evident when she stated that

> *"... if I just write that on the board it could mean anything ... I have got to show them what it is ... so if they understand the method behind what we are doing ... then they will be able to answer future questions ...".*

This is an example of how Maggie used scaffolding to support her learners until they could work independently. After providing sufficient support, she gradually removed the scaffolding. Once Maggie could see that her learners could go on with the problem-solving independently, she stopped illustrating on the board. She also stopped rephrasing students' comments and negotiating meanings. She allowed her learners to continue on their own. This led to their enhanced ability to transfer the acquired rules and knowledge to new activities. This indicated that the learners were now learning at a metacognitive level.

On the other hand, Penny used diverse methods to teach her learners. She believed that, if her learners felt respected and valued, they would participate enthusiastically in the lesson. Penny used lessons focusing on human values as the foundation for her lessons. This was apparent in her statement:

> *"... I am an Education in Human Values facilitator ... we facilitate the values aspect ... we work with tools that will assist pupils ... one of the main tools is breathing ... we encourage them to do deep breathing in order to feed their*

> *brain with some oxygen during the lesson but it also calms them down … it focuses their attention …".*

She preferred to direct her learners' attention so that she could help them explore their understandings. Ollerton (2006) claims that a learner's achievement is linked to a teacher's expectations. In this classroom, Penny expected her learners to participate in her lessons; to engage with the mathematics being taught, and to collaborate with each other in her classroom. These positive notions manifested themselves in her teaching. Her learners were actively encouraged by her attitude to engage with individuals within the learning community in this activity system.

Discussion

Many regard mathematics as a complicated subject. The extent of this complexity envelops both the learning and the teaching of mathematics. This complication presents an obstacle in society, since success in South Africa is generally measured by the amount of mathematics one knows. Thus, it is important to be well equipped with the knowledge of mathematics. To substantiate this point, in order to gain access to higher paying occupations, learners are required to attain a high pass rate in their Grade 12 mathematics examination.

While the effects of being educated during the apartheid era scars many teachers, in the midst of all the politics and bureaucracy some teachers are successful in making a difference in the mathematics classrooms. This success is, in some instances, difficult to explain and warrants exploration. While exploring these success stories, it was evident that learners and teachers interacted with each other in all the classrooms. There was constant meaningful engagement where the teachers used visual tools to scaffold the teaching and learning of mathematics. As discussed earlier, all the master teachers used the CRA approach when teaching mathematics concepts in class.

This study emerged as a result of these success stories, in which the use of visuals was prevalent.. Visuals were used as tools within the mathematics classroom to allow all learners access to the mathematics being taught. The master teachers wanted to concretise the abstract mathematics being taught by using concrete manipulatives and representations before finally proceeding to the abstract level. In some instances, the master teacher in this study used visuals to assist in communicating mathematical ideas when the language of instruction was not the learners' first. In many instances, it served the same purpose as code switching.[1] However, instead of only using language to code-switch, the master teachers used visuals and language. In other instances, when language was not the issue, the master teacher used visuals to clarify concepts being taught.

1 Code switching is the concurrent use of more than one language. It means switching back and forth between two or more languages in the course of a conversation.

Each master teacher used his/her mathematics lessons to make a difference in his/her learners' lives. They were motivated and determined to make a difference. Visuals were used when textbooks, technology and other resources were not available. Master teachers were more interested in the solution process than in focusing on attaining the correct answers. Visuals were also used to remove the abstractness of mathematics. What was new, in this instance, was that each master teacher used the CRA method to scaffold the teaching and learning of mathematics and his/her experience to sequence his/her lessons appropriately. They reflected in and on action while teaching. Once they realised that the learners were not grasping key concepts in mathematics, they re-taught the concepts at a concrete level first, thus allowed learners to manipulate objects, mathematics tools and everyday objects in order to concretise mathematical concepts. This gradually led to the drawing of pictures or the use of colour to represent these concrete mathematical ideas. Once the teacher was satisfied with the progression of the lesson, the mathematical concepts were taught at an abstract level where numbers, symbols and rules were introduced. Each master teacher provided a wealth of thought-provoking strategies. Each activity system encouraged and catered for meaningful teacher-learner interactions. The master teachers behaved as mediators of knowledge rather than as facilitators in the classroom.

This study contributes to the knowledge that these techniques and strategies may be used in any classroom within any social milieu. Apart from poorly resourced schools, these techniques may also be used in schools where the behaviour of learners proves to be the biggest obstacle to learning and where learners are not streamed into ability levels, as is the case in the majority of schools in South Africa.

The scaffolding techniques also proved to be highly effective in large classrooms. In addition, the master teachers demonstrated the usefulness of their visual strategies in large classrooms with limited resources. The master teachers in this study demonstrated that anything is possible if the teacher is determined and committed to making a difference in mathematics education. The master teachers' ongoing professional development also proved to impact positively on how each one of them taught in the classroom.

Essentially, it was apparent in all the classrooms that the master teacher was not the distributor of knowledge, but rather acted as a guide for the educational experience of his/her learners. The role of the master teacher in these classrooms was to help learners identify associations between their unprompted, everyday concepts and the formal concepts of the mathematics discipline. In this way, learners played a more active role in their own learning, and this led to an intrinsic motivation. The learners' enjoyment, fulfilment and interests were emphasised. It was through these observations that the classrooms in this study were regarded as progressive learning spaces rather than traditional teaching spaces.

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