An exploration of the common content knowledge of high school mathematics teachers

Sarah Bansilal

Deonarain Brijlall

Thokozani Mkhwanazi

Many studies point to the problem of poor mathematics content knowledge of mathematics teachers in South Africa. The purpose of this study was to investigate teachers' knowledge of the mathematics they are themselves teaching. Data was generated from the teachers (n = 253) written responses to test that was a shortened form of a previous Grade 12 Mathematics Paper One examination. The sample of teachers were studying towards an Advanced Certificate in Education (an upgrading high school mathematics qualification) at the University of KwaZulu-Natal in South Africa. The findings revealed that the teachers in this sample obtained an average of 57% in the test. Using an APOS theory analysis it was found that many teachers who were working at an action level of a concept would require help and scaffolding to move to process or object levels of understanding of that concept. Furthermore it was found that on average teachers obtained 29% on questions which were at the problem solving level, raising concerns about how these teachers would mediate tasks that are set at high cognitive levels, with their Grade 12 learners.

Keywords: Mathematics teachers, pedagogical content knowledge, common content knowledge, APOS theory, algebra, calculus

Introduction

In South Africa, many studies suggest that mathematics teachers struggle with the content that they teach. In fact, studies point to the teachers' poor content knowledge as one of the reasons for South African learners' poor results in both national and international assessments (CDE, 2011; Mji & Makgato, 2006). Hugo, Wedekind

Sarah Bansilal	Deonarain Brijlall	Thokozani Mkhwanazi
School of Education, University of KwaZulu-Natal E-mail: Bansilals@ukzn.ac.za Telephone: 031 260 3451	Department of Mathematics, Statistics and Physics, Durban University of Technology E-mail: deonarainb@dut.ac.za Telephone: 031 373 2126	School of Education, University of KwaZulu-Natal E-mail: mkhwanazit2@ukzn.ac.za Telephone: 031 260 3548

and Wilson (2010) reported on a study of teaching and learning mathematics in primary schools in KwaZulu-Natal. They found that none of the teachers were able to achieve 100% for the test on the curriculum that they were teaching, and 24% of the respondents got less than 50%. On average, only 47% managed to get each test item correct. Spaull (2011), in his analysis of the Southern and Eastern Africa Consortium for Monitoring Educational Quality (SAQMEQ) 2010 results, revealed that the top 5% of Grade 6 learners (559 students) scored higher marks on the same mathematics test than the bottom 12.5% of Grade 6 educators (62 teachers) in the sample.

Most studies about mathematics teachers' content knowledge in South Africa have been reported in terms of primary school teachers (Hugo et al., 2010; Spaull, 2011) and there are no studies with specific details of what FET mathematics teachers are struggling with. In this study, we attempt to scratch the surface of this issue by investigating the content knowledge of 253 Grade 12 mathematics teachers from various regions in KwaZulu-Natal.

The purpose of this study is to explore the teachers' understanding of the concepts of quadratic equations, patterns, functions (hyperbolic and quadratic), aspects of calculus and linear programming. With this purpose in mind, we formulated the research question: How do Grade 12 mathematics teachers perform on Grade 12 mathematics test items? We hope that the results of this study will add to the knowledge of what mathematics teachers know and understand about the content they teach.

The formal two-year upgrading programme that the teachers were enrolled in was the Advanced Certificate in Education in Mathematics in the Further Education and Training band (ACE: Maths FET). The programme consisted of eight modules, two of which were generic education modules that all ACE students in the Faculty of Education studied. There were four mathematics content modules which focused on probability and statistics; geometry, trigonometry and measurement; differential calculus and integral calculus. These modules were intended to deepen the teachers' content knowledge by including some topics from school mathematics, but also going beyond what was required in the classroom and could be seen as what Ball, Thames and Phelps (2008) call horizon knowledge. This is an 'awareness of how mathematical topics are related over the span of mathematics included in the curriculum' (Ball et al., 2008: 403). Horizon knowledge enables teachers to make decisions about how to teach concepts to their school learners. The programme also included two pedagogic content modules.

Literature review and theoretical framework

Wu (2005: 9) argues that often 'a well-intentioned pedagogical decision in the classroom can be betrayed by faulty content knowledge'; thus, emphasising the importance of content knowledge in the teaching situation. Bansilal (2012b) focused on a teacher's poor mathematics content knowledge and found that the teacher's

explanations were often incoherent and illogical. The teacher's poor understanding of the concepts of ratio and number resulted in her missing some key ideas and presenting convoluted explanations that involved circular reasoning, which made no sense to her learners. In another South African study an author explored links between the pedagogic content knowledge (PCK) and classroom practice in a calculus class at a university (Brijlall, 2011; Brijlall & Isaac, 2011). The data revealed that there was a strong link between PCK and classroom practice.

Many researchers agree that professional development programmes should include a focus on content knowledge and pedagogical content knowledge (Adler, Slominsky & Reed, 2002; Peressini, Borko, Romagnano, Knuth & Willis, 2004; Kriek & Grayson, 2009; Ono & Ferreira, 2010; Bansilal & Rosenberg, 2011). In South Africa, the need for effective professional development of practising teachers was highlighted when many colleges were shut down during the 1990s, while some were incorporated into higher education institutions (HEIs; universities) in 2001. The evidence suggested that many colleges of education were producing teachers of poor quality (NEPI, 1993; Rogan, 2007). The shutting down of the colleges meant that many graduates had to pursue studies at HEIs in order to upgrade their qualifications. This legacy of deep inequality in education provisioning in South Africa has led to a demand for effective professional development programmes. However, coupled with the numerous changes in the school curriculum, professional development programmes often served the triple purpose of upgrading, retraining, and opening up pathways to higher education. Bansilal's study (2012a) showed that teachers who had enrolled for an ACE (Mathematical Literacy) for different purposes had different success rates. Thus, people who enrol for different purposes have different needs, and professional development programmes should take this into consideration.

Are formal professional development programmes the only route for teachers who want to improve their content knowledge? Many researchers believe teachers will only learn if they attend lectures or workshops where they can acquire the required knowledge. However, an important dimension of teacher learning is building up teachers' own mathematical knowledge by reflecting on what takes place in their classrooms. Steinbring (1998: 159) offers a model that provides insight into some of the mechanisms that facilitate the learning of both learners and teachers in the course of a mathematics lesson. While students learn by engaging in a task, interpreting and making sense of their solutions and reflecting on and generalising them, the teacher learns from observing the students' process, varying the learning, and reflecting upon the entire process. Although formal professional development programmes might not be the only route for teachers who want to improve their content knowledge, the focus in this article is on the implications for formal rather than informal programmes.

According to Ball et al. (2008: 395), the term "Mathematical Knowledge for Teaching" refers to 'the mathematical knowledge needed to carry out the work of

teaching mathematics'. The work of Ball et al. (2008) builds upon Shulman's seminal notion of pedagogical content knowledge, which includes:

> *for the most regularly taught topics in one's subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations – in a word, the ways of representing and formulating the subject that make it comprehensible to others. Since there are no single most powerful forms of representation, the teacher must have at hand a veritable armamentarium of alternative forms of representation (Shulman, 1986: 9).*

This description demonstrates the importance of strong mathematical knowledge of 'the most regularly taught topics in one's subject area', because finding ways of formulating the concept so that it is comprehensible to others requires that one must understand the concept itself very well. Ball et al. (2008: 399) refers to 'common content knowledge' as the knowledge of 'a kind used in a wide variety of settings – in other words, not unique to teaching'. Thus, common content knowledge can be seen as the knowledge that teachers themselves mediate with the learners in the class.

In addition to this common content knowledge (mathematical knowledge of the school curriculum), Ball et al. (2008) have provided three other domains of mathematical knowledge for teaching. Specialised content knowledge is mathematical knowledge that teachers use in teaching), while knowledge of students and content is knowledge that combines knowledge of content and students. In this domain, teachers need to be able to anticipate student errors and common misconceptions, interpret students' incomplete thinking, and predict what students are likely to do with specific tasks and what they will find interesting or challenging. Finally, knowledge of teaching and content is knowledge about instructional sequencing of particular content and salient examples for highlighting salient mathematical issues.

In order to carry out the task of teaching, a teacher needs a more profound understanding of the common content knowledge. A teacher should know the various definitions of concepts, be able to select relevant examples and exercises, be able to choose the sequence in treating a specific topic, be able to recognise the usefulness of particular representations over others in certain circumstances, and distinguish between correct and unproductive strategies used by learners when solving problems.

All these tasks of a teacher depend on a robust understanding of the common content knowledge that they are mediating with their learners. In this article, our focus is on the teachers' understanding of this common content knowledge that Grade 12 learners are expected to demonstrate in their final examinations.

Thus, in this situation we view teachers as learners who also require this knowledge. We have argued that this common content knowledge is a necessity for teachers, but it is certainly not sufficient as alluded to by many researchers (Shulman, 1986; Ball et al., 2008; Brijlall, 2011; Bansilal, 2012b).

In order to understand some of the struggles that the teachers have with the school level mathematic concepts that they teach, we draw upon Dubinsky's APOS (action-process-object schema) theory. APOS theory asserts that:

> *An action conception is a transformation of a mathematical object by individuals according to an explicit algorithm which is conceived as externally driven. As individuals reflect on their actions, they can interiorise them into a process. Each step of a transformation may be described or reflected upon without actually performing it. An object conception is constructed when a person reflects on actions applied to a particular process and becomes aware of the process as a totality, or encapsulates it. A mathematical schema is considered as a collection of action, process and object conceptions, and other previously constructed schemas, which are synthesized to form mathematical structures utilized in problem situations (Trigueros & Martinez-Planell, 2010: 5)*

Within APOS theory a person's understanding of a concept is thus seen as transformed from an externally driven entity into a totality upon which other transformations can act. Thus, in learning each concept, such as the parabola, one would start by seeing it in terms of plotting the points and generating a graph from the equation of a quadratic function. Then, as one reflects on the steps, one may be able to generate a graph by working with certain properties such as the turning point, x- and y-intercepts. These smaller algorithms or steps can be seen as part of the overall schema of a quadratic function and these concepts, when encapsulated into objects, can be acted upon in the process of generating a graph of a quadratic function. However, previous non-encapsulations of the prerequisite procedures might hinder a learner from moving to higher levels of understanding of the quadratic function. A written response suggests an action conception when a formula (like the one to solve a quadratic equation) leads to the writer's solving an equation that does not require any rearrangement of the equation, or requires just an input of numbers. We adopt this as an empirical setting for an action formulation since the stimulus (in this case the formula) triggered a response in the writer. This formula can be seen as an external driver. If, however, we observe that the individual can rearrange the given equation into standard form, if necessary, and can compute an expression for the solution if the coefficients are variables, then we may conclude that that individual has displayed a process conception of solving quadratic equations arising from the procedure of using the formula. If, on the other hand, we see evidence in a written response of a conclusion on the nature of roots by interpreting and deducing properties suggested by the expression under the square root sign in the quadratic formula, we can conclude that the individual has displayed an object conception of the quadratic equation because they understand the role and meaning of the constituent elements in the formula. All these notions would provide us with empirical data of the individual's schema for quadratic theory.

Methodology

This qualitative case study employed an interpretative approach, which assumes that people's subjective experiences are real (Cohen, Manion & Morrison, 2007). Denzin and Lincoln (2008: 4) explain: 'qualitative researchers study things in their natural settings, attempting to make sense of, or interpret, phenomena in terms of the meanings people bring to them'. In this study, the phenomenon of interest is the common content knowledge of the group of teachers, which is a dimension of their mathematical knowledge for teaching (Ball et al., 2008). As such, this study is considered to be a case study, which is used if 'you wanted to understand a real-life phenomenon in depth , but such understanding encompassed important contextual conditions — because they were highly pertinent to your phenomenon of study" (Yin, 2009: 18).

The data collection instrument was a shortened form of the National Senior Certificate March 2010 supplementary examination. Owing to time constraints, we shortened the instrument so that the maximum mark was 75 instead of the original 150. The original paper consisted of 12 questions, which we reduced to seven. We removed those questions based on arithmetic and geometric sequences, exponential and cubic functions, financial mathematics, and some minor sub-questions. The questions that appeared in our data collection instrument are summarised in table 1.

Table 1: Details of research instrument

Question	Description	No.of sub-questions	Maximum mark
1	Quadratic equations and inequalities	2	8
2	Patterns	3	9
3	Hyperbolic function	5	11
4	Parabolic function	6	15
5	Finding derivatives using rules	2	6
6	Optimisation	2	8
7	Linear programming	4	18
Total			75

The teachers had two hours to write the test, and wrote under test conditions. Of the group of approximately 350 teachers, 286 agreed to participate in this study.

Two experts marked the teachers' responses and a moderated sample ensured consistency. We captured the marks for each item, after which we removed anomalies and inconsistencies from the cleaned data with respect to missing details or missing records. We ended up analysing 253 records.

Results

In this section, we first present the overall results per question and then we focus on particular trends observed on certain questions.

Overall results

The average mark obtained in this test was 57%. The teachers in this group are all teaching mathematics at FET level, which was a condition of acceptance for the programme. It is disappointing to note such a low average. The following box plot in figure 1 illustrates the distribution of the marks.
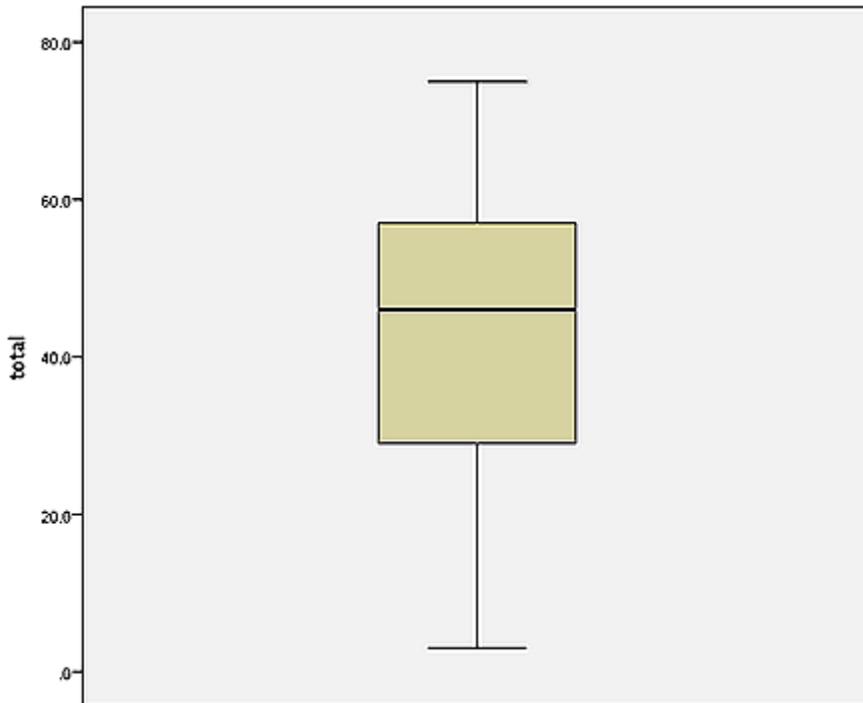

Figure 1: Box plot showing distribution of total marks

Figure 1 illustrates the class median mark of 61% and shows that half of the group got below 61%. The first quartile is at 39%, which means that a quarter of the group obtained below 39% in the test of Grade 12 mathematics. We now look at the results of each section in more detail. Table 2 contains a summary of results per section.

Table 2: Summary of results per question

Question	Description	Average percentage	Number who obtained full marks	Number who obtained 0
1	Quadratic equations and inequalities	75%	112	8
2	Patterns	55%	48	13
3	Hyperbolic function	70%	18	8
4	Parabolic function	49%	32	13
5	Finding derivatives using rules	85%	169	9
6	Optimisation	25%	15	88
7	Linear programming	55%	38	25

The easiest question was that of finding the derivative using rules. Although differentiation in calculus (Q5) was the easiest, the section which required application of the derivative in order to maximise a function (Q6) was experienced as the most challenging in this paper. Thus, the two sections on calculus were on opposite ends of the continuum.

It is also interesting to observe that teachers did better in questions on the hyperbola. On average, they obtained 70% compared to the questions on the parabola, where the average was only 49%.

Also of interest is the fact that teachers did better on questions of finding the derivative than on questions of quadratic equations and inequalities. This result is surprising because the derivative appears only in the Grade 12 curriculum, while quadratic equations are studied in the Grade 10, 11 and 12 curriculums, and quadratic inequalities in the Grade 11 and 12 curriculums. One reason for this could be that, after studying two modules on differential and integral calculus in the ACE programme, using the power rule for finding the derivative became elementary.

We will now briefly look at the breakdown of questions according to the four levels described in DoBE (2011: 53). These levels are L1 (knowledge), L2 (routine procedures), L3 (complex procedures), and L4 (problem solving). These levels account for 20%, 35%, 30% and 15% respectively of formal assessments. Each author separately carried out the categorisation of the questions, after which we compared our results. Three questions were coded differently. We interrogated these items and reached consensus on how to code each of them.

Table 3: Breakdown according to assessment levels

Level	Number of marks per level	Percentage of marks at each level	Average percentage per level
1	7	9%	84%
2	28	37%	73%
3	29	39%	47%
4	11	15%	26%

Table 3 illustrates that, as the cognitive level of the questions increased, teachers did progressively worse. The teachers managed to achieve only an average of 26% in the Level 4 questions. This raised the question of how the teachers who got only 26% (or below) on the problem-solving items would mediate or help their learners to deal with problem-solving questions.

Patterns and equations

Table 4 reveals that the average percentage achieved in solving a quadratic equation using the formula was only 81%.

Table 4: details of results for patterns, quadratic equations and inequalities

Sub-question	Description	Level	Average %	Number who got full marks	Number who got zero
1.1	Quadratic equation using formula	1	81%	179	25
1.2	Quadratic inequality	2	69%	127	26
2.1	Finding fifth term in quadratic sequence	1	93%	235	18
2.2	Finding nth term	2	63%	147	79
2.3	Finding a term greater than the given value	3	40%	50	104

It is concerning that 20 teachers achieved zero for this question. Also disappointing is the fact that the average percentage was 69% for Q1.2 (solving a simple quadratic inequality: $7x2 + 18x - 9 > 0$). Similarly to Q1.1, 26 teachers achieved zero marks for this question. However, 179 and 127 teachers respectively got the solution to the quadratic equation and inequality correct.

Figure 2 provides an example of one teacher's response for Q1.1 This teacher could not recall the quadratic formula correctly. (This is ironic because the formula sheet

that accompanied the test contained the correct formula.) This incorrect formula served as a false stimulus triggering an action that led to a meaningless process.

Thus, a process conception of solving quadratic equations was impeded by an ineffective action level, because the incorrect quadratic formula was used. Even while performing the calculations of multiplied by 0,206, the teacher could not recognise that it was an unfamiliar procedure, as one generally finds the sum and difference in the later stage of the manipulation of the quadratic formula.

Figure 2: A teacher's written response to Q1.1

A teacher's attempt to solve the quadratic inequality (Q1.2) is shown in figure 3. Here, the teacher has factorised the trinomial and retained the 'greater than' sign to assume that the solution is x is greater than both the critical values. In figure 3, the teacher's written response illustrates that a process conception of solving quadratic equations was employed in solving this inequality. Although one needs to solve the associated quadratic equation, this is only done in order to generate the critical values. Then one needs to understand that with an inequality, unlike a quadratic equation where one stops after identifying the roots, it is necessary to find intervals over which the inequality holds. Her previous non-encapsulation of the procedure of solving a quadratic equation did not allow her to adapt her mental structures to accommodate the new situation; she drew upon her process understanding of solving quadratic equations. She did not recognise that the inequality sign required a different interpretation from that of an equality sign as used in solving quadratic equations.

Figure 3: A teacher's written response to Q1.2

In general, for the quadratic inequality, teachers displayed a variety of methods for solving Q1.2. Some used the table method, some used the sketch of a parabola, and some used a number line.

Functions

Here we look at the questions on the hyperbola and parabola in more detail. The teachers found the questions on the parabola harder than the hyperbola, as the average on the parabola was 49% compared to the average of 70% on the hyperbola. Table 5 shows that most teachers did not have problems with working out the x-intercepts of the parabola. They also found it easy to identify the equation of the asymptotes of the hyperbola. Most teachers were able to sketch the graph of the hyperbola, which was encouraging because the graphs involved a vertical shift of 2 downwards and 1 unit to the right

Sub-question	Brief description of items	Level	Average %	Number who got full marks	Number who got zero
Q3	Given				
3.1	Find equations of asymptotes	1	86%	211	27
3.2	Intercepts with axes	2	89%	208	18
3.3	Sketch hyperbola f(x)	2	84%	200	31
3.4	Range of y = –f(x)	3	36%	91	162
3.5	Description of transformation from f to if				
3	20%	27	178		
Q4	Given a sketch of a parabola				
4.1	Find coordinates of one x- intercept	2	90%	226	26
4.2	Find coordinates of second x- intercept	2	80%	199	52
4.3	Find equation in form				
y = a(x − p)2 +q	2	53%	102	74	
4.4	Find equation of reflection of f in x-axis	3	54%	133	118
4.5	Find maximum value of 1 − f(x)	3	35%	77	165
4.6	Solve f(x2 − 2) = 0	3	31%	70	154

Table 5: description of results for items on the hyperbola and parabola

Questions on a slightly higher level, which required a description of the range of the hyperbola, were more challenging. Teachers had difficulty when asked to describe the transformation of f into the given function. The transformation involved a reflection of f about the y-axis (or a combination of a translation and reflection in x-axis). Two teachers' written responses appearing in figure 4 indicate that both the teachers did not seem to understand the question.
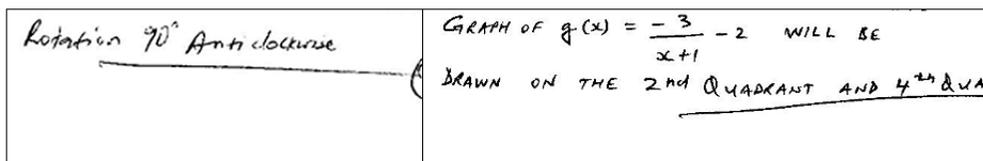

Figure 4: A teacher's written response to Q1.1

In terms of the questions on the parabola, the first two questions did not pose a problem for most. The questions requiring the equation of the parabola posed a challenge for many, with 74 teachers getting zero for this question. It was questions 4.5 and 4.6 that had the worst results. Question 4.5 required a reflection of f(x) across the x-axis and then an upward shift of 1; however, it was necessary to just trace the movement of the one point (turning point), since this point would become the maximum point when reflected. This means that these teachers did not display a meaningful schema for parabolas. Question 4.6 required an algebraic approach, substituting x2 – 2 into f(x) and then solving the equation.

Calculus

In this section, we focus on teachers' responses to two questions where the highest average mark and lowest average mark was attained. Table 6 presents a summary of the teachers' performance, followed by the tasks.

Table 6: Description of results for items on calculus

Sub-question	Level	Average %	Number who got full marks	Number who got zero
5.1	2	90%	214	16
5.2	2	81%	192	34
6.1	3	59%	137	96
6.2	4	13%	16	183

One reason for the high averages (90% and 81% in Q5.1 and 5.2, respectively) in question 5 would be the nature of the questions (find the derivative of two functions), which involved the routine application of the rules for differentiation. These can be described as carrying out procedures which are level 2 questions. In addition, the teachers spent two semesters during the ACE programme studying differential and integral calculus; thus, these questions did not pose a challenge. This level of success

also shows that tasks which require instrumental understanding (rules that can be performed by just knowing the "what" and "how") are more readily taken up than tasks which require relational understanding (understanding the "why" and "when").

APOS theory postulates a leap in understanding from working on an action or process level to working on an object level understanding. This may explain the difference in performance between Q5 and Q6. A recent study (Brijlall & Ndlovu, 2013) on optimisation found a need for an itemised provisional genetic decomposition for individual tasks. For instance, questions 5 and 6 would require more scaffolding to facilitate an easy transition from the action level to the process level for encapsulation into an object. The details of the genetic decomposition of doing this one item at a time will help a teacher provide scaffolding to the learners so that they could be more successful at such tasks. With experience, the teacher will continuously refine the provisional genetic decomposition so that it can diagnose and treat learners' difficulties in this section more accurately.

QUESTION 5 [6 marks]

5.1 Calculate $D_x\left[4 - \dfrac{4}{x^3} - \dfrac{1}{x^4}\right]$. (3)

5.2 Determine $\dfrac{dy}{dx}$ if $y = \left(1 + \sqrt{x}\right)^2$. (3)

QUESTION 6 [8 marks]

A farmer has a piece of land in the shape of a right-angled triangle OMN, as shown in the figure below. He allocates a rectangular piece of land PTOR to his daughter, giving her the freedom to choose P anywhere along the boundary MN. Let $OM = a$, $ON = b$ and $P(x ; y)$ be any point on MN


6.1 Determine an equation of MN in terms of a and b. (2)

6.2 Prove that the daughter's land will have a maximum area if she chooses P at the midpoint of MN. (6)

Figure 5: Questions 5 and 6 of the test

Optimisation is challenging as a question, and can be seen as on level 4 (problem solving) in the assessment taxonomy (DoBE, 2011) because it involves modelling of a real-life situation. Figure 6 is an example of a teacher who tried to find the equation on MN by substituting the y-coordinate of N and the x-coordinate of M into the formula $(y - y1) = m (x - x1)$.

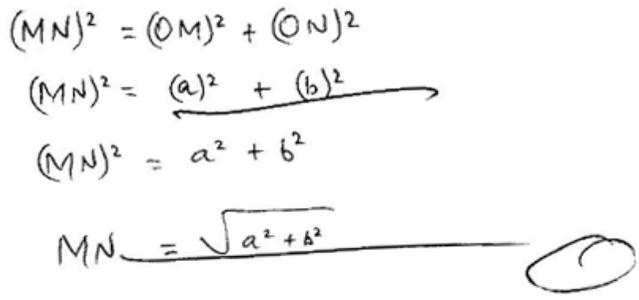6.1   Determine an equation of MN in terms of a and b.
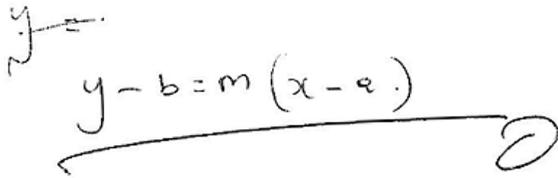
$$y - b = m(x - a)$$

Figure 6: A teacher's written response to finding the equation of MN

Another response (see figure 7) shows that the teacher found the length of line MN in terms of a and b instead of finding a general equation.

6.1   Determine an equation of MN in terms of a and b.

$$(MN)^2 = (OM)^2 + (ON)^2$$
$$(MN)^2 = (a)^2 + (b)^2$$
$$(MN)^2 = a^2 + b^2$$
$$MN = \sqrt{a^2 + b^2}$$

Figure 7: A teacher's written response to finding the length of MN

Discussion and concluding remarks

The results of this study raise concerns about the teaching of mathematics by FET mathematics teachers whose knowledge of school mathematics is so poor.

Teachers need to be able to design valid assessments that can provide a good indication of the current proficiency of particular learners in the subject of mathematics. The Department of Education provides descriptors of four cognitive levels that should be used to guide all assessment tasks (DoBE, 2011). The DoBE (2011: 53) provides the descriptions of the necessary skills for each of the four different cognitive levels. The results of this study (table 2) indicate that teachers performed well at level 1 questions (average of 84%), but as the cognitive level of the questions increased, the teachers' responses were mostly inadequate (average of 47% and 26% for levels 3 and 4 respectively). This renders a somewhat contradictory pedagogic situation. How will teachers design fair assessments that

cover the four taxonomy levels if they are struggling to solve questions based on levels 3 and 4 of the taxonomy? How will they recognise valid alternative solutions to higher-level questions if they cannot produce an appropriate solution themselves?

Across various questions, it was found that some tasks that required an action level of understanding (such as finding the solution to a quadratic equation using the quadratic formula) were within the bounds of most of the teachers' abilities. However, the responses to questions requiring an object level understanding of the quadratic function, such as expressing the quadratic function f(x) in the form y = a(x – p)2 +q (Q4.3), were answered poorly. This shows that the teachers' engagement with the concept of the quadratic function is very low. According to APOS theory, learners who are working at an action level of a concept require help and scaffolding to move to process or object levels of understanding that concept. A teacher could identify a genetic decomposition that would help recognise the level of engagement that particular learners are at and provide opportunities that could lead learners to deeper understanding. This could take the form of explanations, providing consolidation or extended activities, focusing on particular examples, counter examples, or providing different representations of necessary concepts that could help them to deepen their understanding of the concept. These are some of the skills referred to by Shulman (1986) as part of the notion of pedagogical content knowledge. However, all of these interventions would not be available to teachers who themselves are working on an action level of the concept. Firstly, if teachers can work only in an externally driven manner on particular procedures, they will not be able to recognise the demands of the questions and will not have the skills to construct interventions to meet the needs of their learners. The teachers will not have access to various representations of the concept because they are only able to carry out the procedure at an action level. Thus, the teachers' limited understanding is an impediment to the pedagogic content strategies that they will be able to draw upon in the class.

These poor results also call for introspection into the professional development programmes that we have offered. Since the teachers wrote the test in the last semester of their two-year upgrading programme, the results indicate that the programme was not able to develop the teachers' competence in the subject matter that they were required to teach. The results also indicated that teachers found the questions on the parabola harder than the hyperbola – the average on the parabola was 49% compared to an average of 70% on the hyperbola. Initially, this result surprised us. In trying to explain this finding, we scrutinised the content covered in the various modules of the programme and found that the Introductory Calculus module had a strong focus on rational functions, with extensive discussions about limits and asymptotes. The emphasis of the properties and relationships between the variables of rational functions would have helped them to see the shifts and properties of the hyperbola more easily. It was assumed that parabolas were sufficiently covered at school level for many years, so the unit on the parabola was done as a self-study.

Considering that this intervention on rational functions could have made a positive influence on the teachers' competence in hyperbolic graphs, we ask the question:

Should the entire programme have been designed so that all school level content was taught to the teachers? We think that if this had happened, the teachers would have done much better at working out these Grade 12 test items; however, we do not recommend such a step. Providing correct answers to such questions forms a small part of the teachers' task. As research on teachers' pedagogic content knowledge indicates, there is much more than just knowing the common content knowledge (Shulman, 1986; Ball et al. 2008). The results of this study indicate that teachers do not know sufficient school mathematics – which needs to be addressed urgently. Perhaps it should be mandatory for under-qualified teachers to complete a foundation programme that is focused on school level mathematics content before being granted entry into the formal teacher qualification programme of the ACE (or Advanced Certificate in Teaching – ACT) as it will be called in future (DoHET, 2011).

This study has provided information of how practising teachers are struggling with the mathematics content that they are teaching. Although the results are discouraging, it is hoped that the specificity of the results would influence teacher educators and teacher development agencies to urgently design and implement interventions which could be used to improve the situation.

References

Adler J, Slominsky L, & Reed Y 2002. Subject-focused INSET and teachers' conceptual knowledge-in-practice. In J Adler & Y Reed (Eds.), *Challenges of teacher development: An investigation of take-up in South Africa.* Van Schaik Publishers: Pretoria, South Africa, 135-152.

Ball DL, Thames MH & Phelps G 2008. Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5): 389-407.

Bansilal S 2012a. Exploring success rates in a professional development programme for in-service teachers. *Alternations*, 19(2): 236-256.

Bansilal S 2012b. Arriving at the starting point? Exploring a teacher's use of circular reasoning in a mathematics classroom. *Education as change*, 16(1): 35-49.

Bansilal S & Rosenberg T 2011. South African rural teachers' reflections on their problems of practice: Taking modest steps in professional development. *Mathematics Education Research Journal*, 23(2): 107-127

Brijlall D 2011. A case study of lecturers' views on content knowledge and practice. In LT Mamiala (Ed.), *Proceedings of the nineteenth Annual Meeting of the Southern African Association for Research in Mathematics, Science and Technology Education*. North-West University, Mafikeng, South Africa, 34-50.

Brijlall D & Isaac V 2011. Links between content knowledge and practice in a mathematics teacher education course: A case study. *South African Journal of Higher Education*, 25 (4): 660-679.

Brijlall D & Ndlovu Z 2013. Exploring high school learners' mental construction during the solving of optimization. *South African Journal of Education*, 33(2) Art. #679, 18 pages, http://www.sajournalofeducation.co.za.

CDE (Centre for Development and Enterprise) 2011. Value in the classroom. The quantity and quality of South Africa's teachers, CDE in Depth, 11.

Cohen L, Manion L & Morrison K 2007. *Research methods in education*. London: Routledge Falmer.

Denzin NK, & Lincoln Y 2008. Introduction: The discipline and practice of qualitative research. In NK Denzin & Y Lincoln (Eds.), *Strategies of qualitative inquiry*. Los Angeles: Sage, 1-44.

DoBE (Department of Basic Education) (2011). *Curriculum and Assessment Policy Statement (CAPS). Mathematics Grades 10-12.* Pretoria, South Africa.

DoHET (Department of Higher Education and Training) (2011). The minimum requirements for teacher education qualifications. *Government Gazette, no 3446.* Pretoria: DoHET.

Hugo WJM, Wedekind V, Wilson D. et al., (2010). *The state of education in KwaZulu-Natal: A report to the Provincial Treasury*. Pietermaritzburg: KZN Provincial Treasury.

Kriek J & Grayson D 2009. A holistic professional development model for South African physical science teachers. *South African Journal of Education*, 29: 185-203.

Mji A & Makgato M 2006. Factors associated with high school learners' poor performance: a spotlight on mathematics and physical science. *South African Journal of Education,* 26(2): 253-266.

NEPI (National Education Policy Investigation) 1993. The framework report and final report summaries. Cape Town: Oxford/NECC.

Ono Y & Ferreira J 2010. A case study of continuing teacher professional development through lesson study in South Africa. *South African Journal of Education*, 30: 59-74.

Peressini D, Borko H, Romagnano L, Knuth E & Willis C 2004. A conceptual framework for learning to teach secondary Mathematics: A situative perspective. *Educational Studies in Mathematics*, 56(1): 67-96.

Rogan JM 2007. An uncertain harvest: A case study of implementation of innovation. *Journal of Curriculum Studies*, 39(1): 97-121.

Shulman LS 1986. Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2): 4-14.

Spaull N 2011. *A preliminary analysis of SACMEQ III South Africa*. Stellenbosch: University of Stellenbosch.

Steinbring H 1998. Elements of epistemological knowledge for mathematics teachers. *Journal of Mathematics Teacher Education*, 1(2): 157-189.

Trigueros M & Martínez-Planell R 2009. Geometrical representations in the learning of two-variable functions. *Educational Studies in Mathematics*, 73: 3-19.

Wu H 2005. *Must content dictate pedagogy in mathematics education?* Talk given at California State University, USA.

Yin RK 2009. *Case study research: Design and methods*. Beverley Hills, CA: Sage.