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Focusing on the object of learning and what is critical for learning: A case study of teachers' inquiry into teaching and learning mathematics

Ulla Runesson

During the last decade, Lesson Study, a Japanese professional development model involving a community of learners, has been spreading to countries around the world. Lesson Study is a systematic process of inquiry into classroom practices where teachers collaborate in planning, implementing, observing and revising lessons. The approach used in the study reported in this paper involves a version of Lesson Study, namely Learning Study. Whereas the underpinning theory in Lesson Study is often implicit, Learning Study has an explicit theory that can help teachers to pedagogically theorise about their practice, students' learning and the object of learning. Through one Swedish Learning Study in mathematics, I demonstrate what teachers can learn by exploring what is critical for their students' learning and how teaching can be improved to enhance learning.

Keywords: Lesson Study, Learning Study, teaching algebra, teacher learning, variation theory

It has been argued that teachers' professional learning must build on interaction and collegial learning in professional learning communities (PLCs) (e.g. Borko, 2004; Cochran-Smith & Little, 1999). In PLCs, teachers' learning is directly related to and embedded in their work, deriving from teacher experiences in relation to the specific context in which they work and the learners they encounter daily. Similarly, following Dewey's model for 'learning to teach', Elliott (2012) argues that it is important for teachers to get opportunities to inquire into teaching and learning using a systematic approach, where the classroom serves as a laboratory. The classroom teaching and learning, therefore, become a source for collecting data that can be analysed critically and reflected upon. Van Driel and Berry (2012: 26) have noted concerns, however, that PLCs 'tend to ignore issues related to teaching and learning subject matter'. I share these concerns, particularly from the point of view of current discussions

Ulla Runesson
Jönköping University, Sweden & University of the Witwatersrand
E-mail: Ulla.Runesson@hlk.hj.se
Telephone: +46 36101361

regarding the importance of teachers developing sufficient knowledge to teach the subject effectively (Pedagogical Content Knowledge, PCK). I suggest that the PLC approach should include opportunities for teachers to develop their knowledge of students' learning and, at the same time, increase their knowledge on how to teach their specific subject matter effectively. What and how teachers can learn from a PLC approach, with emphasis on specific subject matter and student learning, will be described and discussed. Below, I describe a group of primary teachers' collective learning experience as they worked with converting a sentence into an algebraic expression, for example, a sandwich costs 2 dollars more than a Coke or x+2=y. Particular attention will be paid to the process by which teachers surmise and predict the critical aspects of the above algebraic expression and how teaching strategies are reshaped to account for newly surmised critical aspects.

Learning Study: A systematic inquiry based on a pedagogical theory

The approach used was Learning Study, a version of the Japanese Lesson Study (Lewis, 2002; Stigler & Hiebert, 1999; Yoshida & Fernandez, 2004). Just like Lesson Study, Learning Study is a kind of teachers' collective exploration of classroom practice. It contains a jointly planned lesson, observation of the enacted lesson, and a revision of the lesson plan based on reflection on the lesson, usually in a cyclical sequence. However, compared to Lesson Study, which can have various objects of enquiry (Lewis, 2009), Learning Study always has the object of learning as the focus. This is a consequence of the underpinning pedagogical theory, namely variation theory. (For a more detailed account see, e.g. Marton & Booth, 1997; Marton, Runesson & Tsui, 2004.) One element of variation theory is that learning is seen as the learning of something; learning always has an object. From a variation theory perspective, learning primarily involves a qualitative change in the way the object is experienced. The way an object is experienced can be defined in terms of the aspects that are focused on and discerned simultaneously. In other words, for learning to take place, learners must be able to discern aspects of an object they were not able to discern before. For every object of learning (the capability we want the learners to develop), there are certain critical aspects. What the critical aspects might be, cannot be derived from the subject matter alone, but from the learners, their difficulties and their ways of experiencing what is learned. Teachers can conjecture on what the critical aspects might be and, by putting them to the test in their practice, confirm or reject them as being critical. For example, it could be that an important aspect of the subject matter has already been discerned by learners and thus need not be featured as a critical aspect within the current focus of teaching. Consequently, critical aspects

figure 'relationally' – as a function of both the subject matter and the focal learner group.

Discernment comes from being able to make distinctions: to see differences and recognise those aspects that are necessary to discern. For example, to discern an aspect of an object, it is necessary to see it as an aspect that can vary (a dimension of variation). 'Colour' is an aspect that can vary (different colours); 'shape' is another. To be able to discern a certain aspect, the learner has to experience a variation in that aspect. To discern the aspect 'colour', it is necessary to have experienced at least two different colours. Without having experienced at least two different shapes, it is unlikely that the aspect 'shape' will be discerned.

This principle can guide teachers when they plan for helping learners to discern the critical aspects. According to Bowden and Marton (1998), that which varies against a stable background is likely to be discerned. By creating a pattern of invariance and variation (e.g. showing five cups of the same size, shape and material, but with different colours), it is likely that the varying aspect (colour) will be attended to and, thus, discerned.

Learning Study draws upon the variation in the way both students and teachers see and deal with the object of learning, while being guided by the aforementioned principles from variation theory (the object of learning and its critical aspects, and experienced variation as a prerequisite for discernment). The various ways in which students' perceive the object of learning are regarded for distinguishing what is critical (i.e. what is not yet discerned). In the same way, the sharing of variation in the teachers' knowledge and experience (often tacit or unnoticed) provides valuable input into Learning Study (Pang & Lo, 2012). This sharing is an important aspect in identifying what is critical for learning and how to make this learnable in the classroom.

To summarise, the aim of the collective inquiry into teaching and learning is not to develop teaching practice in general (e.g. to make the lessons more interactive or implement new teaching strategies and technologies), but to gain insights into the relationship between the subject matter, how it is taught, and student learning, with a particular focus on a specific object of learning and its critical aspects. It includes systematic data collection and collective analysis of teaching and students' learning in terms of what learners must connect, differentiate and be aware of. Furthermore, the teacher must determine how this could be made possible in the lesson.

The cyclic process and the purpose with each step are described in figure 1. After having decided on the object of learning, the teachers conjecture what the critical aspects for learning may be. This phase is based on previous teaching experience and a diagnostic 'test' comprising several tasks they believe can reveal learners' difficulties and conceptions. The aim of this test is to confirm and/or identify critical aspects to be taught in the lesson. Based on an analysis of how the learners solve the tasks, the first lesson in the cycle is jointly planned with a particular focus on how

to teach the critical aspects. Next, one of the teachers implements the lesson(s) in her class. From an evaluation of the learning outcomes (how the learners solved the same set of tasks after the lesson), together with close observation of the recorded lesson(s), conjectures about the critical aspects and how to teach them to make them discernible are discussed and confirmed or rejected, and a subsequent revised lesson plan is produced. A second teacher uses this revised lesson plan with her class, and this continues until all the teachers in the group have taught the lesson(s). One Learning Study process usually takes approximately three months.

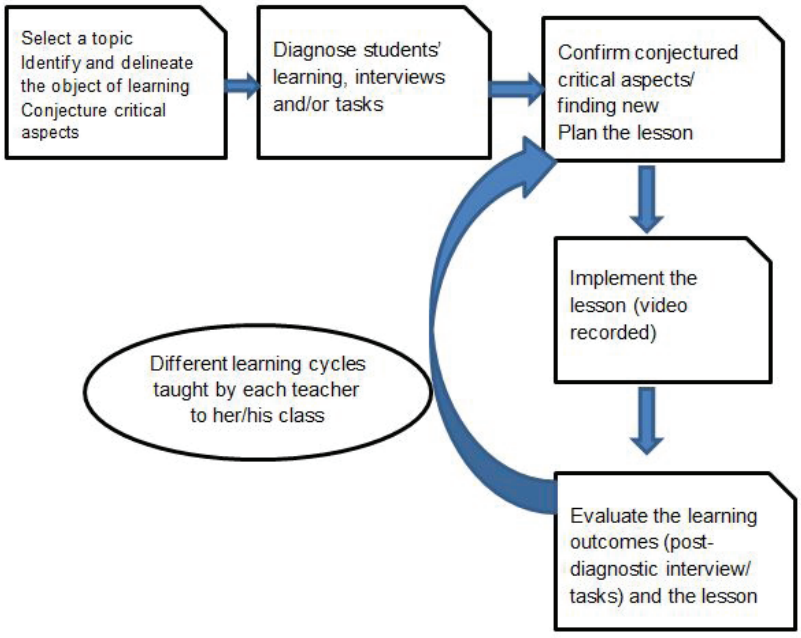

Figure 1: The Learning Study cycle

The study

In the following sections, I demonstrate how this systematic inquiry into classroom practice can be a site for teachers to obtain insight into their assumptions about their students' learning, and how their taken-for-granted ideas about how to facilitate learning and how to teach a particular lesson can evolve.

The research context and participants

A group of three Year-4 and -5 teachers in a Swedish school in a suburban area of a large city worked together with two researchers (the author and a research assistant) from the local university. They took part on a voluntary basis and were selected for this study because the researchers were aware of their interest in school development. The teachers were familiar with variation theory because they had attended a

one-day seminar held at the university and had participated in two previous Learning Studies. They were all primary school teachers and were, therefore, general teachers rather than subject experts, with between 5 and more than 30 years of teaching experience. They taught their own classes and were familiar with the students. With a few exceptions, the majority of the students had a Swedish ethnic background and the medium of instruction was Swedish. The classes were all mixed-ability groups consisting of 25, 27 and 23 learners respectively. All students were 11 years old. Participation was optional for the learners. The parent(s) of the participants had given their written consent. The ethical guidelines given by the Swedish Research Council were followed (information, confidentiality, consent and voluntariness).

Data construction and data analysis

Data for this article included video-recordings and full, verbatim transcripts from the six lessons (in total 240 minutes) and (partially transcribed) audio-recorded meetings with the teachers before, during and after the completion of the lessons (in total six meetings, each one approximately 90 minutes). Initially, I listened several times to the audio recordings, and noted instances that were interesting from the perspective of the aim of this study. All of these were transcribed verbatim.

While analysing the data from the teacher meetings, I listened carefully to the audio recording and followed the discussion in order to ascertain the aspects that the teachers commented on in the recorded lesson (although talk from the lesson they were watching could be overheard). By simultaneously reading the time-coded transcripts from the lessons and the recording from the planning/evaluation sessions, I was able to interpret the instances in the lesson they commented on, even though they were not talking in full sentences and often referred to 'this' or 'there', for instance: 'There I could have given an example that was incorrect,' or 'Did you write X and Y there?' In the analysis, I worked with the (full) audio recordings, the (partial) transcripts of these and transcripts from the lessons in parallel. The focus was on how the teachers expressed their ideas about enhancing student learning in terms of the following: what they assumed to be critical; how their understanding of critical features changed on the basis of their observation of the recorded lessons; the analysis of pupil learning outcomes of the diagnostic pre- and post-test; and how the lessons were taught.

Exploring teaching and learning of converting sentences into algebraic expressions

In the first meeting, the teachers decided on the capability that they wanted the student to develop (i.e. the object of learning). From the results of the National Assessment Test, they noticed that the learners had difficulties with converting sentences into algebraic expressions (c.f. Booth, 1984; Kieran, 2004; Knuth, Alibali,

McNeil, Weinberg & Stephens, 2005). For example, transforming a relationship such as 'an ice-cream costs 5 kronor more than a coke' into an algebraic expression' (i.e. x+5=y) confused the learners, possibly because there was no calculation needed. One teacher commented: 'They are used to always doing a calculation, so they do not know what to do'. So, the point of departure was the identified difficulties and, from this, the aim was to explore why they were problematic and how teaching could help the students to overcome these learning problems. This is a specific feature of Learning Study: the object of learning is chosen from teachers' experience of what they have found to be difficult to learn. To gain deeper understanding of these difficulties, the teachers designed the diagnostic test that they hoped would reveal specific problems the learners experienced. From a variation theory perspective, learners' difficulties with learning and the variation between different ways of understanding the same thing, can be due to learners not having 'discerned' the aspects that need to be discerned. The diagnostic pre-test was designed by the teachers and comprised tasks they believed would give more information about the learning difficulties. In this way, they identified what learners had not discerned yet. This test was taken by all the students in the three classes.

Conjecturing the critical aspects

The teachers used an interpretative approach to analyse the results of the diagnostic test. The total score was not the main interest, but rather how the tasks were interpreted and solved by the learners.

They found the results from two of the items on the diagnostic pre-test particularly informative. One of the tasks entailed matching a sentence with an algebraic expression. In the sentence, '3 pens and 1 book costs as much as 5 pens', students were given a choice between four algebraic expressions, where two were correct. Only 12 of 47 pupils were able to find the two correct expressions.

The other valuable sources of information were the responses to the open question: 'Why do we sometimes use letters in mathematics?' Only seven out of 47 students answered in a way that suggested they could conceptualise letters as substitutes for numbers (e.g. 'There could be a figure instead and you have to find the number', 'a secret number', 'a short-hand for a number'). A majority gave answers which the teachers interpreted as letters relating to the cognitive demand of mathematics: pupils expressed that letters either make mathematics easier ('because it is easier to understand') or more complicated ('so you have to think really hard') or even that letters have a purpose of their own ('so there is something to learn').

In this discussion, two conjectures arose. One was that the letter representing the variable might be problematic, particularly the letters X and Y. One teacher noted:

T3: If we use X and Y, it will be frightening to the student.

Therefore, they conjectured that, in order for the learners to understand that a letter represents a variable, it would help if the letter chosen was as 'close' as possible to

the variable by using abbreviations. T1 gave an example of how a sentence could be represented algebraically in the lesson:

> *T1: Let's take 'John is three years older than David'. This we could write 'John's age – David's age = 3 years'... and next, shorten this to 'J–D=3'.*
>
> *T2: Yes, that would be one way.*

So, instead of using X and Y as representing the variables, it was suggested to use abbreviations that were close to what the variable represented (J=John's age and D=David's age).

It must be noted that teachers have ownership of a Learning Study; they are allowed to decide on the object of learning and how to teach. This implies unpredictability and management dilemmas (Adamson & Walker, 2011). However, Fernandez, Cannon and Chokski (2003: 182) conclude that the collaboration 'must include room for knowledgeable coaches who can stimulate the thinking of groups so they can rise beyond their own limitations into rich arenas'. We (the author and the research assistant) took this advice seriously and, even though we were doubtful about some of the teachers' suggestions, we did not reject their ideas. Instead, teachers were asked to continually justify their decisions and we encouraged them to come up with varying alternatives. We conceived of ideas as well, but only as alternatives, not as the experts' 'truth'. Our aspiration was to create an atmosphere of open inquiry and debate, a conversation with argumentation, while maintaining a critical approach to the process.

The second conjecture the teachers had was that, in order to see the relational character of the expression (i.e. there is a relation between John's and David's age, in the example above), they must be able to discern the various ways in which this can be described (verbally and mathematically). They, therefore, decided to demonstrate a variation of the expression, for instance: 'David is three years younger than John', 'The difference between their age is three years' and the corresponding algebraic expressions (e.g. 'J–D=3; J–3=D; D+3=J; J=D+3; D=J–3; 3+D=J'). The equations are all permutations of D+3=J, that is, the position of the symbols and, thus, the operation changes whereas the variables are the same. This pattern of variation they planned to apply to several examples.

Only after decisions were made on how to work with the content in the lesson, were the particular teaching arrangements, organisation and resources determined. This is also a principle from variation theory; that teaching what is critical for learning is significant, and that solely taking a general approach to teaching (e.g. whether the learners should work in groups, what material to use) does not provide the necessary conditions of learning. Instead, the specific conditions in terms of aspects of that which is learned play the most significant role in making the learning of the object of learning possible.

They agreed on the structure of the lesson:

1. Introduction (whole class): converting a sentence into an algebraic expression and presenting all the possible permutations (several examples).
2. Activity in pairs (worksheet): learners should find the algebraic expressions and all the permutations to each sentence and vice versa.
3. Learners at the blackboard (whole class): accounting and arguing for their solutions on the tasks.

This structure was used in all three lessons. The first lesson was taught in accordance with the planning by teacher 1 to her class.

While planning the first lesson, what were the views of the teachers regarding how to best teach the object of learning in order to enhance learning? One aspect they highlighted was 'to see the relational character of the expression'. When they designed the teaching of this aspect, they drew on principles from variation theory; the same example was used, whereas the positions of the variables (and the operations) varied. The lesson was designed also to teach the 'representation of the variables'. Their assumption was that the abstract letters X and Y were the problem and they decided to avoid them by using other letters. Instead of having X and Y invariant in the different expressions, they decided to vary them depending on the variable they represented.

Revising and rejecting the critical aspects

As explained previously, the main aim of a Learning Study is to enhance students' learning by ascertaining the critical aspects which might make a difference in learning. Therefore, it is important for teachers to study the lesson, the learners' reaction to the teaching, and their ways of dealing with tasks given after the lesson (usually the same as before the lesson). The analysis of this diagnostic post-test implies that teaching be related to learning, not as a one-to-one correspondence, but finding out whether the topic was dealt with in a way that made it possible to learn what was intended and what must be changed for the next lesson. With the background of the particular tasks on the post-test the learners still had difficulties with, the teachers carefully observed the recorded lessons. This inquiry into teaching and learning allowed them to consider how appropriate their teaching strategies in the lessons actually were.

In the post-lesson meetings following the first lesson, new conjectures were made about aspects that must be made learnable in lesson 2, and previous assumptions were reconsidered and even rejected.

In the post-lesson meeting after lesson 1, a new conjecture was devised: to understand that the difference between the variables is the same, independent of the values taken. This was initiated by T1 when she observed herself teaching. In the lesson, she gave an example of the relationship of age between two persons.

Different permutations of one expression describing the relationship were presented. She commented on this by saying:

> *T1: [in a low voice] But … it doesn't matter if I am 60 [years old] or 20! He is always 5 years younger … I don't know if that came up in the lesson, did it?*

My interpretation is that T1 noticed a feature of the relationship she had not noticed before. However, nobody answered the question on that occasion and the discussion led into other issues while they were attending to the recording. Later on, when they talked about using the same example (about comparing age) in lesson 2, my interpretation is that the idea was obtained. One teacher said:

> *T3: Change their ages!*

Even though they did not explicitly say that this was necessary to learn, it was taught in lesson 2. The teacher showed the same example (invariant) and varied the numerical value of the variables. In this way, the learners would be able to realise that the difference in age between John and David is the same, independent of their age at a particular point in time.

Then they decided to bring out an important feature of the relational character of the expression; not all permutations correspond to the example. This came from reflections on the result on the diagnostic post-test together with their observations in the recorded lesson. It was noticed that many of the learners could still not choose the two correct algebraic expressions for the sentence: '3 pens and 1 book costs as much as 5 pens'. In the lesson, the teacher asked whether the examples she gave corresponded with the expression, even though all the examples were correct. She said:

> *T: Does that [example] correspond with the formula?*

Questions such as these or 'Is it correct?' were used repeatedly throughout the lesson. The teacher who had conducted the lesson became aware of the character of the questions she asked in the lesson when she observed the recording:

> *T1: Here [in this situation] I could have given an example that was incorrect. Yes, I could have come up with that, one that was wrong!*
>
> *Researcher: Yes, writing an incorrect one!*
>
> *T1: Yes, I could. They should have had [an incorrect] one. That's why they failed on that [test] item [in the diagnostic post-test]. That's why they haven't got any further.*
>
> *T2: Yes, when going through the first example you could have chosen …*
>
> *T1: … I could have given the example 'l+7=9 [and asked] is that correct'?*

This excerpt suggests that she could see a connection between what happened in the lesson and the results on the post-diagnostic test: 'That's why they have failed', she said. She could also see a possible solution to this: presenting an incorrect expression.

The excerpt illustrates how they became aware that only 'correct' expressions had been used through carefully watching the recorded lesson and questioning what they had previously taken for granted. Using incorrect examples had not occurred to them when planning the first lesson, but was now seen as a possibility. Consequently, an incorrect expression was added to the previously planned examples. For example, D–3=J was used with the example above as an incorrect expression that maintained the relational character of the algebraic expression, while varying the relation itself.

As a result, the second lesson taught by T2 in his class, was taught in accordance with this planning, except for one unexpected suggestion by the learners, namely that other letters (X and Y, A, B, C and symbols such as square and circles) be used for the variables. When the teacher asked for a formula for the example given, one pupil suggested using X and Y.

Christian: You can write X minus ten equals Y!

T2 writes X–10=Y on the board.

T2: Explain. How did you work this out?

Christian: Mm, X that's you [your age] minus ten years and Y is Johan.

Although the teacher wrote this on the board (X–10=Y), he did not pick this up at all. Instead he said:

T: Okay, So you changed our names to X and Y instead. Is it possible to change my name to something else besides X and Y? Elsa?

Elsa: Er ... to letters.

T2: Yeah, what letters?

Elsa: A or B, C.

T2: Yes, what else?

S?: (inaudible)

T2: Yes. Fredrika?

Fredrika: If you knew the age, you could take Johan's age plus ten equals yours.

This episode was observed in the post-lesson meeting. For instance, T1 expressed her awareness by asking:

T1: Did you write X and Y there [later on in the lesson]?

T2: No, I didn't. I wanted to get to the abbreviations H and J. Therefore, I didn't want to confuse them with X and Y. I didn't want to use that.

When noticing how the learners responded, they realised that some of their assumptions about the learners' problems were not totally correct. The learners (at least several in this class) seemed to understand that any letter could be used as a representation, so using abbreviations might not be a good thing. Therefore, they decided to deliberately introduce other symbols than abbreviations in lesson 3, but keep to the planning to show incorrect algebraic expressions and substitute the

values of the variables to emphasise the relational character of the expression. This was also taught in the third lesson.

To summarise, the teachers had certain conjectures about how to learn and teach an algebraic expression, which were rejected, changed or refined. The conjecture that using the symbols X and Y was the problem was rejected when it became apparent that the learners could easily use any symbol for the variable. In this case, it appears that the teachers had underestimated the learners' abilities and they realised that their conclusions about the reasons for the learners' problems from the diagnostic pre-test may have been incorrect, thus, they changed the strategy in lesson 3. However, conjectures were not only rejected, but also further developed. The second conjecture that 'the relational character of the expression could be made learnable by opening up for a variation of the relation between the variables' was kept throughout all the lessons, but with minor enhancements in the two last lessons. They thought that the relational character could be more explicitly exposed by using incorrect expressions together with the substitution of different numerical values for the letters.

Teachers' learning from the Learning Study

The results of the data analysis pointed to a collective process of open and critical scrutiny into teaching and learning. Furthermore, the study entailed a bottom-up approach where the teachers 'owned' the object of inquiry. In other words, the teachers did not implement tasks or activities produced by experts but had to develop alternatives on their own, making choices independently, and seeing where the different choices might lead them. The process was close to and embedded in their practice and related to their professional task. The content, the learners' difficulties and how to teach algebraic expression were the focus. However, I would also suggest that the teachers had to deal with learning and teaching in a specific way. They explored the conditions for learning, not as general learning conditions, but specific to the object of learning in terms of what must be made possible to learn in order for their learners to develop the targeted capability. It has been demonstrated how the critical aspects emerged through the Learning Study, not from the content alone, but through a combination of the learners' understanding together with the teachers' instruction.

Marton et al. (2004) suggested that Learning Study entails learning on three levels: students' learning, teachers' learning and researcher's learning. The data in this study only allowed me to establish what was made possible to learn, not what was actually learned. One thing made possible to learn was that the teachers' assumptions about students' learning could sometimes be incorrect. For instance, the assumption that the representations X and Y were frightening and unknown to the students had to be rejected. This allowed them to understand the importance of being sensitive to the students' perception of the object of learning and, consequently, adapting the teaching to their level of understanding.

Further, it is likely that the teachers' came to realise that some of the taken-for-granted ideas for facilitating learning may not have the expected outcome (i.e. avoiding the letters X and Y was not an effective strategy). At the very least, teachers were provided with the opportunity to see alternative ways of teaching a particular content and, moreover, how lessons can be changed to maximise targeted objects of learning.

Variation theory offers the opportunity to focus on the object of learning and to find answers to the question: 'What is critical to the object of learning?' To some extent, the teachers in the study succeeded in finding answers to this question. From the diagnostic pre-test, they found that very few students connected letters in mathematics with numbers. Therefore, the teachers surmised that the introduction of the X and Y factors would be confusing to the students and was deemed a critical aspect (although this was later dismissed).

Another potential critical aspect that was discovered was the relational character of the algebraic expression. This implied that they identified features of the relationship as being critical for the learning of the object of learning, for example, an algebraic expression could be permutated in multiple ways but it will still describe the same relationship, and the relationship is the same regardless of the values of the variables.

Nuthall (2004) argues that teachers need an explanatory framework in which to understand how their actions affect students' learning. In Learning Study, variation theory is used for this purpose. It has been demonstrated that it adds value to Lesson Study in that respect (Pang & Marton, 2005; Pang & Lo, 2012).

It is not possible to draw far-reaching conclusions about the significance of the use of variation theory from this study, since this was not the aim. However, in my opinion, if subject matter is to play a larger role in teachers' learning in PLC, and if we are going to follow Nuthall's (ibid.) advice, a theory for the learning of the content will likely contribute to more efficient learning. Otherwise, there may be a risk that the focus will fall solely on general aspects of learning, as Van Driel and Berry (2012) argue. This is not to say that variation theory is the only possible theory.

Implementing Lesson and Learning Study

Learning Study, and similar teacher research groups, has contributed considerably to the education systems of Japan and China. Stigler and Hiebert (1999) give credit to the Lesson Study approach as contributing to Japanese students' excellent performance on international tests. In China, Ma (1999) attributes elementary teachers' profound understanding of fundamental mathematics to their involvement in teaching research activities, which are quite similar to Learning Study. China has a long tradition of teachers engaging in teacher research groups similar to Lesson Study (Yang & Ricks, 2012). The strength of teachers' instruction in Chinese mathematics and their profound understanding of fundamental mathematics – despite elementary teachers not having university degrees – has been explained by their involvement in teaching

research activities and in school-based networks of collective learning (Ma, 1999). It appears that Learning Study not only helps students achieve high test results, but also aids teachers' in acquiring knowledge of subject matter. In China, at least, it seems as if such research activities can compensate for a lack of formal education.

If Lesson Study can lead to better student learning, it follows that engaging teachers in Lesson Study approaches will likely assist in achieving educational goals. Ono and Ferreira (2010) report on an initiative to introduce Lesson Study as a model of in-service training for teachers of mathematics and science in South Africa. They point to several challenges. One is that teachers must change focus from covering the curricula to learners' understanding of the content. Doig and Groves (2011: 89) suggest that a crowded curriculum might also be a constraint, since it gives fewer opportunities for a 'longer-term, deeper study of a more limited number of mathematical topics'. Another challenge concerns time. Lesson and Learning Study take place over a period of time; it is therefore necessary to secure time for regular meetings (Ono & Ferreira, 2010). Despite obvious concerns as to whether Lesson Study is possible outside the collaborative school cultures of Japan and China, research (e.g. Andrew, 2012; Davies & Dunnill, 2008; Lewis, 2009; Perry & Lewis, 2009; Yoshida, 2012) has demonstrated that Lesson Study can, in fact, be sustained in other counties and be applied to different subjects and levels of education.

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