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Teachers' understanding of mathematical cognition in childhood: Towards a shift in pedagogical content knowledge?

Elizabeth Henning

This article about the discourse of pedagogy as related to child cognition in mathematics addresses the issue of what constitutes the main disciplinary content and the pedagogical content knowledge (PCK) of foundation-phase teachers. I argue that, unless child cognition itself is the primary disciplinary content of foundation-phase teachers' knowledge, it is likely that they will couch their pedagogical knowledge in teaching methods and materials more than in knowledge of conceptual development of learners and how such knowledge relates to teaching. In this first of a series of case studies, workshop-generated conversational and interview data were analysed qualitatively for discourse. The topics for the workshops were mathematical cognition and training in standardised test administration. The analysis showed that the discourse of teachers' expressed knowledge about their practice was embedded in the language of policy, curriculum, teaching methods of mathematics, and the omniscience of the annual national assessments (ANAs) in South Africa, with very few discourse markers representing knowledge of child cognition. During the course of the intervention, teachers gradually shifted their talk, expressing some understanding of trends in contemporary developmental cognitive psychology and neuroscience of mathematical cognition. The article recommends a stronger cognitive science focus in teacher professional development initiatives.

Keywords: Educational neuroscience, pedagogical content knowledge, mathematical cognition, childhood education, teacher education, elementary school education, foundation-phase education, conceptual change, teacher development, constructivism.

Elizabeth Henning
UJ Institute for Childhood Education,
University of Johannesburg
E-mail: ehenning@uj.ac.za
Telephone: 011 559 5104/2

Background: Teachers learning about mathematical cognition

This study of teacher development forms part of a longitudinal programme of research of children's mathematical and arithmetical competence in the foundation phase (elementary school). The participating teachers learn to administer a mathematical competence test and are also introduced to the conceptual model that informs the test (Fritz, Ricken, Balzer, Willmes & Leutner, under review; Ricken, Fritz & Balzer, 2011). At the same time, they learn relevant aspects of cognitive developmental psychology and educational neuroscience. This diagnostic instrument is used to assess 4- to 8-year-old children and is a local version of a European standardised test that is currently being translated, piloted and standardised in four South African languages with over 800 children participating. In the data from 249 tests administered over the first two years of the pilot project in 2011 and 2012, some trends have been identified that relate to the administrators/testers themselves (Dampier & Mawila, 2012; Henning, 2012a). Nine administrators are also teachers in a university teaching school, where their development as mathematics educators includes this training. The article focuses on these teachers and how they responded to the training in test administration and related topics that emanated from the training, asking the question: How does teachers' discourse on their professional practice and knowledge shift during test administration training and related professional development workshops on the mathematical cognition of children? The study is a descriptive case study that portrays what may be regarded as teachers' emergent conceptual change as evident in their discourse change.

The genesis of the case study was teachers' participation in training for test administration. To understand the thinking of testers, their assumptions about mathematical cognition were recorded prior to the first training session in which they learned how to administer the 45-minute diagnostic interview. In addition to nine administrator trainees who are foundation-phase teachers, 27 are senior foundation-phase education undergraduate students at a university, and 10 are fieldworkers from an educational research agency. Upon closer examination of the administrators' conceptions of child cognition, we found that they gave scant attention to ideas about learning in childhood. We regarded the teachers' discourse with concern and thus investigated the issue with pre-existing data from teacher development workshops on child cognition, which I had been running for a few months as part of the larger research and development project.

Shulman (1986) proposed that, in addition to subject (disciplinary) content and general pedagogy and educational knowledge, teachers should develop an integrated epistemology and concomitant practice that exemplify specific ways to communicate subject content knowledge. Foundation-phase teachers, however, are teachers of all subjects in the curriculum and are thus not subject teachers of one subject (Merseth, 2012). Their pedagogical content knowledge (PCK) is thus different to that of subject specialists, because it is more general, incorporating the whole curriculum. I argue that their speciality is the developing child, specifically the child who is developing cognitively-emotionally in the areas of language, literacy, science and mathematics. They are thus, on this view, specialists of child development/learning itself, and their practice of teaching should mirror an understanding of the developing/learning child (across the curriculum). For the topic of this article, this means that they are specialists in knowledge about the child learning and forming concepts of numeracy and of other components of mathematics in the first instance. Hence, I would argue that foundation-phase teachers need more content knowledge about child cognition than about pedagogy. Inserted in their learning of mathematics, language, literacy, science and art for foundation-phase teaching, should be the latest research on children's learning. In our research group, we work with the longer term hypothesis that their teaching will increasingly become more learning-centred as they understand more about mathematical cognition as researched in both the behavioural and the educational neuroscience fields (Posner, 2010; Sousa, 2010). Unfortunately, textbooks for teacher development are not always the sources of such knowledge. (See the textbook by Montague-Smith & Price [2012] as an example and very different one by Schwartz [2008].)

In the remainder of this article, I shall first set out the analytical framework for investigating teachers' change of epistemological position with regard to PCK. I used conceptual change theory, following Carey (2009), as an analytical lens to capture teachers' shifting understanding of children's mathematical cognition and how this shift may begin to feature in their discourse on classroom practice. I shall refer only briefly to the conceptual model of mathematical cognition (Fritz et al., in press; Ricken et al., 2011)) that the test designers used to inform the instrument which the teachers were learning to use. The methods of the inquiry and the discussion of the findings will follow, concluding that learning to use a test such as the MARKO-D, accompanied by professional development training in some introductory aspects of the cognitive developmental psychology (and some educational neuroscience) of mathematics learning, may be a fruitful avenue for changing the knowledge and the epistemological position of teachers and that this will be observable in their discourse. To offset the article, I shall briefly discuss what I regard as the dominant pedagogical discourse in foundation-phase education in South Africa, namely constructivism.

Constructivist pedagogy as the dominant discourse

Although teachers are generally aware of constructivism as an epistemology, this knowledge is often directly, and I would argue, somewhat thoughtlessly, recontextualised (Bernstein, 1996) as a specific type of pedagogy with defining characteristics. At the turn of the century, Phillips (2000: 1) warned:

> *'Constructivism' is a currently fashionable word in the Western intellectual firmament, one which has beguiled a great many educational researchers, curriculum developers, trainers of teachers and teachers themselves … The philosopher, Michael Devitt, nominates constructivism as a candidate or the 'most dangerous contemporary intellectual tendency' … while Renders Duit regards it as 'a fashionable and fruitful paradigm'.*

In the wake of what became known as 'constructivist pedagogies', deployed in 'the constructivist classroom', the recontextualisation (Bernstein, 1996) of the epistemology in the empirical world of educational practice took on a whole new life form. What is essentially a philosophy of knowledge (Piaget & Garcia, 1991) and of knowing (Von Glasersfeld, 1995) became an empirical workhorse and, if I may play with the metaphor, also a clothes horse for teaching methods and techniques.

Critical discourse analyst Norman Fairclough (2003) shows how a dominant discourse (such as the philosophy of constructivism) feeds off its wide adoption and creates a new social reality (such as constructivist pedagogies), from which the discourse then feeds in a cycle of repetition until the idea becomes so prevalent that it is regarded as a characteristic of empirical reality instead of a lens through which to gaze upon reality. I am of the opinion that this is what happened to the work of Piaget in some areas of education. For general consumption in non-specialised courses and in popular workshops, his massive oeuvre of 88 books and hundreds of articles was reduced to textbook simplicity of how to teach according to his theories. The generic texts contained mostly only his ideas about stages of cognitive development, reference to assimilation and accommodation of knowledge, and some reference to disequilibrium. The context of his work as epistemology was replaced by such textbook generalisations that often failed to distinguish between empirical behaviour and ideas about behaviour. Teacher education programmes are just too cluttered to allow for in-depth studies of such a proliferous scholar and so a simplified and reduced version of this huge body of knowledge was shrunk to byte sizes to fit teacher education curricula. Von Glasersfeld (1995: 53) writes:

> *Those who venture to summarise Piaget's ideas on the basis of two or three of his books have a limited perspective ... At best they provide an incomplete view of Piaget's theory, at worst they perpetuate distortions of his key concepts.*

Furthermore, Piaget's work has been (and I would say, wrongly) juxtaposed with the ideas of his contemporary, Lev Vygotsky (Kozulin, 1990), who published much less in his short life. Many teachers in South Africa, as elsewhere, were educated to offset Piagetian constructivism against the presumed counter-discourse developed from the early harvesting of English translations of Vygotsky's two books, written in the 1930s (Vygotsky, 1962; 1978). The first English translations of his work became totems of matters 'social' and 'cultural' in learning and were recontextualised in matters of teaching. More or less similar discourse events ensued, as was the case with Piaget's work. Vygotsky's ideas were reduced and made palatable, divorced from the context of Stalinist Russia. What is more, the two scholars were placed on opposing sides of the epistemological and pedagogical spectra.

This brings me to the question of how teachers of young children, educated with the type of general integration of epistemology and pedagogy, as described in this section of the article, form what Carey (2009) refers to as a 'conceptual system' of knowledge for teaching. I reason that, if teachers are introduced to this type of epistemological thinking, it may have a persistent influence on how they view pedagogy. Easy-to-use textbook ideas of constructivism may form the foundation for their tacit theories of learning, thinking, for example, that young children are able to learn concepts only in specific (Piagetian) stages, not realising that evolution has given us some innate knowledge of number (Carey, 2009; Wynn, 1992; Dehaene, 2011). They may also, for example, not know that mathematics is a recent cultural artefact, developed by human beings, and relying heavily on language and other symbols for its initial mastery. They may not know that they have to teach very systematically and use language very deliberately and selectively as main instructional medium. They may not know how to create what Venkat (2013) describes as 'connections' in lessons. I believe that contemporary psychology and neuroscience research give us some of the information that may help teachers to build what I will refer to as 'a PCK conceptual system', which is aligned with scientific knowledge of the past two decades and not with the constructivist pedagogies of (often outdated) textbooks.

Teachers' 'conceptual systems' of PCK

To clarify what is meant by a conceptual system, I refer to Susan Carey's model of conceptual systems (Carey, 2009). I use it not to refer to children's systems of concepts, but to teachers' thinking about teaching and learning. On this model, a conceptual system is a collection of concepts that form a framework or a blueprint for understanding phenomena and directing action.

In such a conceptual system, the knowledge referents, the discourse and the broader semiotics are intertwined and, although not cemented, they do remain somewhat closed (Figure 1 presents two such hypothetical systems that illustrate conceptual change).

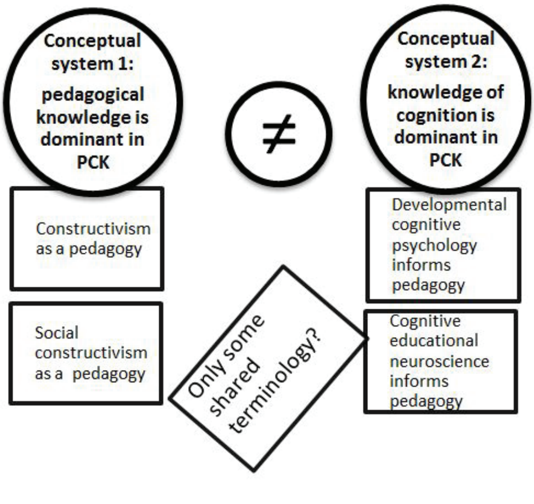

Figure1: The discourse of two hypothetical conceptual systems of PCK

The ideas comprising a new conceptual system are, furthermore, not commensurable with ideas and discourse of an antecedent system. Such systems contain not only different concepts, but also concepts that "are incoherent from the point of view from the other" (Carey, 2009: 359). In explaining this, Carey (2009: 371-376) refers to the history of physical theories such as thermal theory and oxygen theory. Once science had progressed beyond the original theories, it was no longer possible to think of the subsequent theories (lodged in conceptual systems) in terms of the first ones. For instance, I argue that, if a teacher knows what neuroimaging tells us about learning (even though it may still be only a little), s/he will think differently about how to teach, although, agreeing with Coch (2010: 139), this does not mean that there are "easy-to-follow recipes for classroom practice", based on educational neuroscience.

For example, if one knows that time, space and number areas of the brain are physically proximal (Gallistel, 2012), one will understand more fully why prepositions are such gremlins in word problems. Such a realisation may have a direct impact on the way in which we wish to teach sequencing of numbers and how we will teach prepositions as well as the adverbs of time and place. In a (hypothetical) PCK conceptual system that is embedded in this type of knowledge, it would be difficult for a teacher not to be very careful with the way she expresses concepts in language. In other words, a teacher will be more aware of his/her communication and other discourse strategies. S/he will not necessarily change his/her method or strategy itself.

Changing a concept about learning/teaching requires not only new information, although information is vital to form concepts to constitute a conceptual system (CS). It is also not about changing a belief system only.

Changing a conceptual system within which one's knowledge is lodged is no easy task, especially if it can impact something as personal as teaching, where experience and habit together have created a personal belief of what works in a classroom. It is unlikely that any teacher will set out to deliberately change a conceptual system within which s/he conducts his/her work. If one agrees with Snow, Griffin and Burns's (2005) notion of the development of professional teacher knowledge(s) through stages of learning and practice as a teacher, there is little doubt that change does not come easily to teachers' personal belief and conceptual systems of pedagogy.

South African teachers have been subjected to three curriculum revisions in little over a decade. In each of those three transitions, they were expected to adapt their practice in some way. I am not sure if teachers managed the conceptual changes required for these three educational curriculum shifts, not because they do not have the ability, but because they do not have sufficient knowledge content to inform their change and shake their beliefs. It was expected of teachers to develop a 'PCK conceptual system' for which they did not have the tools. Some of these tools can be found in recent research on learning.

A teacher development intervention: Conceptual development in mathematics based on current knowledge

The topics for the workshops and the training session, from which data for this inquiry were sourced, were introduced with the aim of 'bootstrapping' (Carey, 2009) or supporting teachers' change of concepts of what constitutes mathematics pedagogy by changing their views of learning. The workshops were organised to include the following topics:

- Core knowledge of number.
- An overview of the history of mathematics as cultural artefact.
- Hierarchical development of mathematical concepts in early childhood (the MARKO-D conceptual model).
- Conceptual development and linguistic representation.
- Educational neuroscience and behavioural learning mathematics.
- The MARKO-D test and the constructs that each of the 55 items measures.

In all five meetings, the emphasis was on current developmental cognitive psychology and educational neuroscience. The reasoning was that this type of knowledge may achieve what curriculum directives find difficult to do, namely change teacher thinking. The interesting scientific facts in themselves can appeal to teachers' epistemological and pedagogical beliefs. As early as 1983, Hart argued that "teaching without an awareness of how the brain learns is like designing a glove without a sense of what a hand looks like" (Sousa, 2010: 13). With current interest in the work of scholars such as Gallistel and Gelman (1978), Wynn (1992), Dehaene (2011), Butterworth (1999; 2005), Goswami (2008), Sousa (2010), Posner (2010) and other prominent developmental cognitive psychologists and linguists, Hart's assertion may be a sign of what will inform education in future. In his recent books on the neuroscience of reading and on mathematics learning, Dehaene (2009: 228; Dehaene, 2011: 275) sets out what some of the implications of new neurological research may be for education. For example, he argues that, if knowledge of the structure of DNA could change our understanding in many areas of medical science and change practice, knowledge of the physiology of the brain may have a role to play in our conceptions of literacy education and mathematics teaching.

I would add that, combined with results from three decades of cognitive developmental psychology in this field, change in the teaching of foundation-phase mathematics may come about more easily through knowledge of child cognition (and its biology) than through endless workshops on methods of teaching. Teachers can be shown a landscape of psychology and neuroscience portrayed by researchers such as Le Corre, Van de Walle, Brannon and Carey (2006), Wynn (1992), Dehaene and Brannon (2011) and Xu, Spelke and Goddard (2005) and examine their own practice with a different lens. The type of lens is captured in the international educational organisation, Mind Brain and Education (Sousa, 2010; Fischer, 2010), which strives to bring scientific knowledge of the brain to the classroom, while critiquing the 'neuromyths' that abounded in the popular literature in the 1990s.

When I set out to capture teachers' discourse on their practice and their thinking, I was hoping to find some indication of emergent conceptual change about their practice (and thus what feeds their practice, which, I argue, is their PCK). I aimed to do this through the exemplar of mathematics teaching, my reasoning having been that this specific form of pedagogy requires a very specific understanding of learning.

The inquiry

The inquiry is a descriptive case study of teachers' discourse within the bounded system of their conversation in four workshops, a training session and four individual interviews within a five-month period, with a follow-up group interview after eight months. The boundaries of the case (Stake, 2005; Henning, Van Rensburg & Smit, 2004) were thus very specific. The ethical clearance for this research was obtained in the programme of research of the Education Faculty of the university, in which teachers agreed to the research on condition that they and their place of work will remain anonymous. They signed an agreement with the university upon their appointment as teachers at the school, in which they made themselves available for research.

Methods

Data sourced from workshop discussions and interviews

The process of the inquiry is represented in Figure 2, showing the sequence of

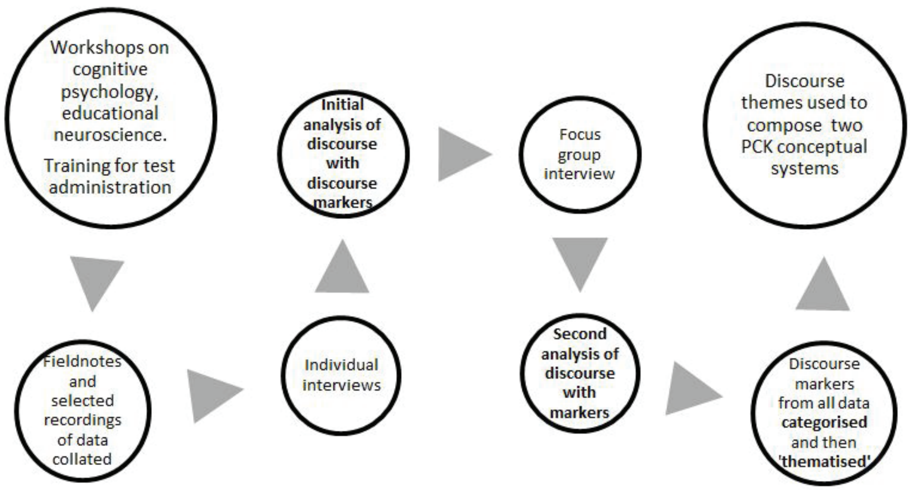

Figure 2: The process of data collection and analysis

In the teacher development workshops, the discussions took place in three formats:

- The conceptual model for the MARKO-D was introduced along with introductory ideas of psychology and neuroscience, with a demonstration and discussions of a computer game, The Number Race, designed by Stanislas Dehaene and Anna Wilson (Dehaene, 2011: 281).
- The teachers watched brief extracts of video recordings of their teaching in their school, after which the extracts were discussed from the perspective of the theoretical knowledge to which the teachers had been introduced.
- The development work culminated in the training to use the MARKO-D test.

I made notes during and after the workshop sessions and audio recorded only pertinent and ethically permissible sections of the first and the last one, amounting to 95 minutes of audio recordings. I also recorded interviews with four of the teachers on their views of teaching prior to the workshops and the training. During the revision of this article, I conducted a focus group interview with eight teachers and the school principal. The data sources constitute a descriptive collage of teacher discourse over a period of eight months, captured in five conversations and four individual and one group interview(s).

Discourse markers for data analysis

The different data sets were collated and analysed in two phases, both of which were aimed at identifying discourse markers (Figure 2). In this process I marked/labelled utterances with descriptors such as "specific teaching method", "education department quality assurance", "curriculum compliance", "the brain", "annual assessments", and so forth.

I first used only field notes and audio recordings (without transcriptions, in order to pick up on paralinguistic phenomena, such as tone and volume, and also in the interest of time). Upon the recommendation of reviewers of the work, I employed the services of a transcriber for all the audio data and repeated the labelling of discourse markers, selecting utterances (beyond single words) that occurred repeatedly. The dominant discourse markers (45 in total) were then grouped in the following categories:

Initial discourse

1. Teaching mathematics consists of the use of methods (believed to have been derived from constructivism).
2. Teaching/instruction of mathematics is minimised by classroom management needs.
3. Compliance to the national curriculum is the object of teaching

mathematics.

4. Expectations of the provincial education department regarding administration in the classroom are high.
5. ‘CAPS and ANA training’ (national curriculum and annual assessments) are important components of their work duties as teachers of mathematics.
6. Teachers find it difficult to provide support for children who struggle with number.
7. Issues concerning language in the classroom, referring mostly to the school’s policy of translation and dual language education in mathematics lessons.

Emergent discourse

8. Language is the mediator of learning mathematics.
9. Children learn mathematics by forming numerical and other mathematical concepts.
10. Children learn mathematics by repetition, feedback and practice.

From these categories, I deduced the following overall themes of the teachers’ discourse: teaching has become a bureaucratic function for the teachers; they are keen to learn about learning, as was evident in the emergent discourse to which I refer briefly in the findings; they are somewhat fixated on methods as beacons of expertise, a theme I identified from the grounded analysis of the data, and they are finding it difficult to linguistically explain the concepts that they try to teach. Categories 8-10 were marked by lexical items that were prominent in the training workshops, including the use of terms such as “neurons”, “language areas”, “prefrontal cortex”, “symbolic representation”, and “number sense”.

Findings: A keen interest and some change

Bearing in mind the categories and the themes, I composed two “conceptual systems”. The first one, Conceptual System 1, has the qualifier “PCK with curriculum compliance in mind”. From a much less frequent set of utterances from only 11 original discourse markers out of a total of 45, I composed Conceptual System 2, with the qualifier “Emergent PCK with learning in mind”. The majority of the markers used as indicators for Conceptual System 1 contained terms and phrases such as “rhythmic counting”, “doing the bonds”, “using group-work”, “counting in twos/threes”, “they must count every day to warm up”, “we use the number charts for everything”, “no fingers!”, “sharing is fractions”, “estimate is how many there are”, “they must write the numbers neatly”, “they must know their number line”, “I explain everything with writing on the board”, “how are they going to do this on the ANAs”, “I assess to see if they do it right", "it's what we learned in CAPS training", "I have not read the CAPS completely".

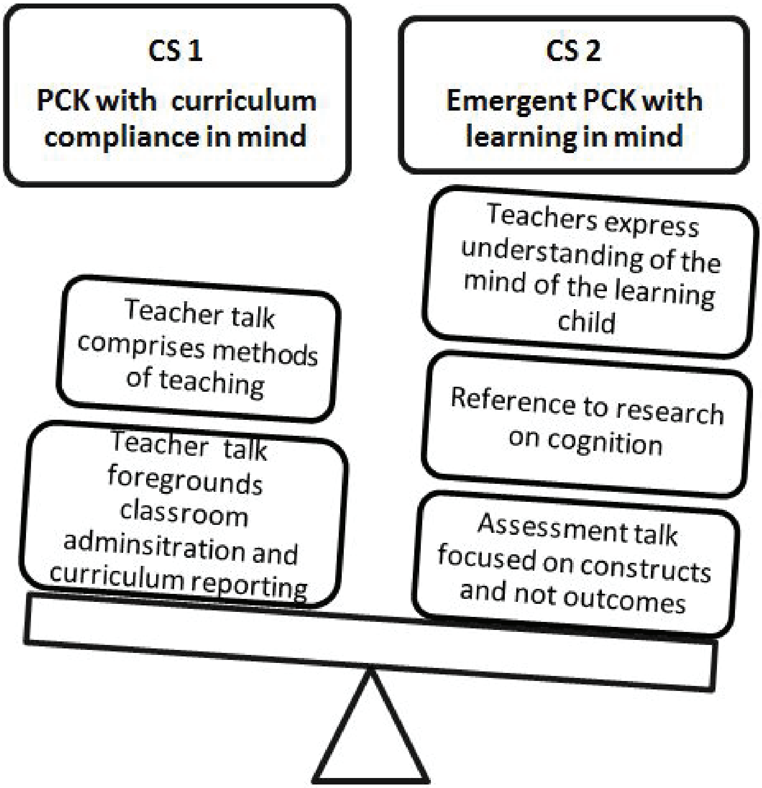

Figure 3: Emergent epistemological shift in teachers' discourse

In the interest of space, instead of citing selected verbatim utterances, I shall summarise the main trends in the data. By far the most dominant theme was the teachers' continuous conversation about the national curriculum, with policy issues and administrative tasks and assessment comprising most of the talk. Throughout the conversation, the curriculum manifested consistently, even though it was not at all the direct object of conversation at every new conversational turn. References to specific aspects, including page numbers of the document of the Curriculum and Assessment Policy Statement were more frequent than any other reference, with the discussion of the ANAs a close second. Another dominant theme was the teachers' reference to the use of material resources and different methods and techniques that they had learned and of which their classrooms exhibited ample evidence. They were also concerned about the language of instruction, about code-switching and in which language the learners were to be tested in the ANAs. Their talk about child cognition was minimal and vague at the outset and, as expected, comprised the need to practise a constructivist pedagogy and to "use cooperative" learning techniques, especially when they noticed a learner who struggled. They also noted that rote learning was not beneficial. They emphasised group work and related various techniques of "doing group work", ensuring that "social learning takes place, because children learn from each other".

However, after the second workshop, the teachers started to ask questions about mathematical cognition. I noted that the neuroscience information awakened more interest than the psychology and asked them why the brain was of more interest to them than the psychological experiments of conceptual development which we had discussed. Only two teachers volunteered to respond. They admitted to being attracted to the fact that one could see the products of the fMRI scans and the other scans. One teacher mentioned that she now thinks that she will understand how her children speak and act, regarding them as people "with a fast moving brain", instead of learners who struggle to calculate. They were surprised to learn that brain activity can be viewed in milliseconds and that areas of activity can be pinpointed. The brief discussion we had on core cognition/knowledge of mathematics also became a focus. I had the sense that the teachers had never really thought of the physiology of learning or of evolution as theoretical context for studying learning. The leading converser in the group said as much:

> *I always thought learning was a bit magic. We as teachers have to let them do the work and then the rest is just magic. If they do the work they will learn … Sometimes the magic does not happen and then we have to do more work, and they have to do more work until they get it right. If they don't get it right second time I know they are just a slow learner.*

Another teacher, who had been reluctant to speak, said:

> *I just think of me and how I must finish this teaching and mark the books and be a good teacher. I knew that some children struggle and that some others are fast, and I try to help them, but I never ever thought of the detail.*

There is an ironical twist in these findings, as the teachers' discourse during the training and development sessions was recognisably camouflaged by a 'learner-centred' lexicon with a strong emphasis on constructivism as pedagogy. Yet, they hardly mentioned child cognition itself. The most talked about social issues related to the pupils and how children "learn together". It perturbed me that they did not refer to children's talk or questions. Talk about this was, to a large extent, absent. The object of their pedagogy discourse was not the learning child.

Discussion and conclusions: Child cognition as core of PCK for foundation-phase teachers?

What is evident from this brief foray into the participating teachers' talk about young children's conceptual development in mathematics is that it had never been an object of much reflection for them and, if it had been, they did not refer to it. Their epistemological-pedagogical position, which I wished to describe, remains, at best, adrift. The data show clearly that they view themselves, with their methods and techniques, and leaning on the school curriculum, as somewhat mechanical conduits for child learning, gleaning skills and knowledge from whatever source comes their way. However, they do not appear to know themselves as professionals – what makes them good teachers of mathematics for young children, as described by Thames and Ball (2010). Judging by some of the comments, they have no shortage of workshops, meetings and newsletters to inform them as to how to be a good teacher. Two of them served as "CAPS trainers" for the local education department. By and large, I am of the opinion that they are regarded as good, effective teachers. There is no question that they work very hard.

I have no single way of interpreting the findings, even though I make the effort to regard the findings as a manifestation of (if only barely emergent) conceptual change (Figures 1 and 3). In this instance, I need to emphasise that the work is exploratory and that it is an exercise in getting to know the field of teacher talk (and behaviour) concerning foundation-phase mathematics teaching. The article can claim no more than that. The encouraging signs about conceptual change in a series of ongoing workshops beg for more research. At least the teachers have some information about recent research and it did set them onto some new discussions. Towards the end of the workshop series, there was an indication that they are beginning to wonder about mathematical cognition and about the model that we had presented to them.

Thinking from the perspective of Shulman's (1986) notion of different teacher knowledge(s), I do draw one conclusion: In this case study, the teachers' PCK, as expressed in their talk, comprises mostly general pedagogical knowledge, with only glimpses of subject-specific pedagogical knowing, as a way of knowing – "fachspezifisch-pädagogischen Wissen" (Lange et al., 2011). The content knowledge of childhood mathematical cognition is still foreign to the teachers in this research sample. Their response to the introduction of the topic was, however, very positive.

Based on the participants' observable interest in the model and the underlying knowledge bases of cognitive developmental psychology and neuroscience, I make a case for PCK of foundation-phase teachers to include as much knowledge as possible of recent research on mathematical cognition of young children. I agree with Ball et al. (2008) that Shulman's PCK model is not a fixed model. Perhaps it is time to consider a different variant of it for teacher education and development in the foundation phase. Thames and Ball (2010) conclude that, besides knowledge and skill, there also needs to be fluency in knowing mathematics for teaching. I argue that such a fluency (coherence) may be found in an understanding of the developing mind of the child, coupled with what mathematics, as a cultural phenomenon, has to offer young children (Dehaene, 2011; Dehaene & Brannon, 2011; Brannon, 2002; Butterworth, 2005).

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78827) and from the Zenex Foundation. The views expressed in this article are the author's only.

Endnotes

i) In referring to cognition, the role of emotion and motivation is included.

ii) MARKO-D is an acronym, translated from German as "mathematical and arithmetical competence diagnostic". The test (Fritz et al., 2013) is available from the publisher, Hogrefe Verlag, Göttingen.

iii) His work on children's development of number is an exception, because the users are usually mathematics education scholars who refer to his work in a more specific fashion (Piaget, 1965, for example).

References

Ball DL, Thames MH & Phelps G 2008. Content knowledge for teaching: What makes it special? *Journal of Teacher Education,* 59(5): 389-407.

Bernstein, B 1996. *Pedagogy, symbolic control and identity*. London: Taylor & Francis.

Butterworth B 1999. *The mathematical brain*. London: Macmillan.

Butterworth B 2005. The development of arithmetical abilities. *Journal of Child Psychology and Psychiatry,* 46*:* 3-18.

Coch D 2010. Constructing a reading brain. In A Sousa (ed), *Mind, brain and education. Neuroscience implications for the classroom* (pp.139-162). Bloomington, IN: Solution Tree Press.

Dampier GA & Mawila D 2012. Test items and translation: Testing mathematical concepts reliably? *South African Journal of Childhood Education*, 2(2): 35-57.

Dehaene S 2009. *Reading in the brain. The new science of how we read*. London: Penguin.

Dehaene S 2011. *The number sense. How the mind creates mathematics.* 2nd ed. Cambridge: Oxford University Press.

Fairclough N 2003. *Analysing discourse. Textual analysis for social research*. New York: Routledge.

Fennema E, Franke M & Carpenter TP 1993. Using children's knowledge in mathematical instruction. *American Educational Research Journal,* 30(3): 555-583.

Fritz A, Ehlert A & Balzer L (in press). Development of mathematical concepts as basis for an elaborated mathematical understanding. *South African Journal of Childhood Education,* 3(1).

Fritz A, Ehlert A, Ricken G & Balzer L 2013. *Mathematik- und Rechenkonzepte für die 1. Klasse - Diagnose*. (Mathematical and calculation concepts for the first grade. Diagnostic.) Göttingen: Hogrefe.

Fritz A, Ricken G, Balzer L, Willmes K & Leutner D (in press). Key numerical concepts at pre-school and early primary-school age: An integrative five-level model of cumulative arithmetic skills development. *Early Childhood Research Quarterly*.

Gallistel CR 2012. Mental magnitudes. In S Dehaene and E Brannon (eds), *Space, time and number in the brain. Searching for the foundations of mathematical thought* (pp.3-12). Amsterdam: Elsevier.

Gopnik A & Meltzoff AN 1997. *Words, thoughts, and theories*. Cambridge, MA: MIT Press.

Goswami U 2008. *Cognitive development. The learning brain.* New York: Psychology Press.

Henning E 2012a. Research in language and conceptual development in the foundation phase. Invited keynote address presented at the conference on *Early Childhood Development, Numeracy Education*. Rhodes University, 9-12 September.

Henning E 2012b. Learning concepts, language, and literacy in hybrid linguistic codes: The multilingual maze of urban Grade 1 classrooms in South Africa*. Perspectives in Education,* 30(3): 69-77.

Henning E, Van Rensburg W & Smit B 2004. *Finding your way in qualitative research.* Pretoria: Van Schaik.

Kozulin A 1990. *Vygotsky's psychology. A biography of ideas*. Cambridge, MA: Harvard University Press.

Lange K, Kleickmann T & Möller K 2011. Elementary teachers' pedagogical content knowledge and student achievement in science education. ESERA - Conference, Lyon, France, 5-9 September.

Langhorst P, Ehlert A & Fritz A 2012. Non-numerical and numerical understanding of the part-whole concept of children aged 4 to 8 in word problems. *Journal für Mathematik-Didaktik* 33(2): 233-262.

Le Corre M, Van de Walle G, Brannon EM & Carey S 2006. Re-visiting the competence/performance debate in the acquisition of the counting principles. *Cognitive Psychology,* 52(2): 130-169.

Merseth K 2012. *New teacher knowledge and the PCK of elementary school teachers*. Guest lecture at the University of Johannesburg, 17April.

Phillips DC (ed) 2000. *Constructivism in education. Opinions and second opinions on controversial issues*. Part 1. Chicago: University of Chicago Press.

Piaget J 1965/1941. *The child's conception of number*. New York: Norton.

Piaget J & Garcia R 1991. *Toward a logic of meanings*. Hillsdale, NJ: Lawrence Erlbaum.

Posner, MI 2010.Neuroimaging tools and the evolution of educational neuroscience. In A Sousa (ed), *Mind, brain and education. Neuroscience implications for the classroom* (27-44.). Bloomington, IN: Solution Tree Press.

Resnick LB 1989. Developing mathematical knowledge. *American Psychologist,* 44: 162-169.

Ricken G, Fritz A & Balzer L 2011. Mathematik und Rechnen – Test zur Erfassung von Konzepten im Vorschulalter (MARKO-D) – ein Beispiel für einen niveauorientierten Ansatz. (Mathematic and arithmetic – Assessment of math concepts in pre-school children – an example for a level-oriented test.) *Empirische Sonderpädagogik,* 3: 256-271.

Schwartz JE 2008. *Elementary mathematics pedagogical content knowledge: Powerful ideas for teachers.* Boston: Pearson - Allyn Bacon.

Shulman LS 1986. Those who understand: Knowledge growth in teaching*. Educational Researcher*, 15(2): 4-14.

Snow C, Griffin P & Burns MS 2005. *Knowledge to support the teaching of reading. Preparing teachers for a changing world.* San Francisco: Jossey-Bass.

Sousa A 2010. How science met pedagogy. In A Sousa (ed), *Mind, brain and education. Neuroscience implications for the classroom* (9-26). Bloomington, IN: Solution Tree Press.

Spelke E 2000. Core knowledge. *American Psychologist*, 55(11): 1233-1243.

Stake RE 2005. Qualitative case studies. In NK Denzin & YS Lincoln (eds), *The Sage handbook of qualitative research.* 3rd ed. (pp. 433-466). Thousand Oaks, CA: Sage.

Thames MH & Ball DL 2010. What mathematical knowledge does teaching require? Knowing mathematics in and for teaching. *Teaching Children Mathematics,* 17(4): 220-225.

Venkat H 2013. Using temporal range to theorise early number learning in South Africa. *For the Learning of Mathematics*, 33(2): 31-37.

Von Glasersfeld E 1995. *Radical constructivism: A way of knowing and learning*. Washington DC: The Falmer Press.

Vygotsky LS 1978. *Mind in society*. *The development of higher psychological processes.* M Cole, V John Steiner, S Scribner & E Souberman (eds and translators). Cambridge, MA: Harvard University Press.

Vygotsky L 1992. *Thought and language*. Cambridge, MA: MIT Press.

Wynn K 1990. Children's understanding of counting. *Cognition,* 36: 155-193.

Wynn K 1992. Addition and subtraction by human infants. *Nature,* 358: 749-750.

Xu F, Spelke ES & Goddard S 2005. Number sense in human infants. *Developmental Science,* 8: 88-101.