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Big Ideas in primary mathematics: Issues and directions

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This article is located within the literature arguing for attention to Big Ideas in teaching and learning mathematics for understanding. The focus is on surveying the literature of Big Ideas and clarifying what might constitute Big Ideas in the primary Mathematics Curriculum based on both theoretical and pragmatic considerations. This is complemented by an analysis of the evidence for two Big Ideas in South Africa's Curriculum and Assessment Policy Statements for Foundation- and Intermediate-Phase Mathematics. This analysis reveals that, while there is some evidence of implicit attention to Big Ideas in the Curriculum, without more explicit attention to these, teachers and, consequently, learners are not likely to develop understanding of Big Ideas and how they connect aspects of mathematics together.

Keywords: primary mathematics, big ideas, CAPS, Foundation, Intermediate, curriculum, South Africa.

Introduction

It is accepted that learning Mathematics is richer, deeper and longer lasting when children make connections between different mathematical ideas (see, for example, Hiebert et al., 1997). Some even argue that "the degree of understanding is determined by the number and strength of the connections" (Hiebert & Carpenter, 1992: 67). Learners connecting different mathematical ideas could be said to be engaged in developing networks of Big Ideas.

While writers argue for a focus on 'Big Ideas' in learning and teaching mathematics (see, for example, Charles, 2005; Siemon, Bleckly & Neal, 2012), there is no agreement about what actually constitutes a Big Idea.

This article surveys the literature on Big Ideas and argues for a definition that encompasses ideas being both mathematically and conceptually 'big', and embraces pragmatic and pedagogical considerations. The South African Curriculum Assessment and Policy Statements for Foundation Phase and Intermediate Phase (Department for Basic Education, 2011a; Department for Basic Education, 2011b, henceforth

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referred to as CAPS FP and CAPS IP) are examined for evidence, explicit and implicit, of two specific Big Ideas. On the basis of this analysis, I argue that a lack of explicit attention to Big Ideas in these South African curriculum documents is a limitation that could contribute to a fragmented knowledge of mathematics and, consequently, be a factor in the continued low attainment in mathematics in South Africa. Finally, I consider some teaching consequences for developing Big Ideas in mathematics in primary school.

What is a Big Idea in mathematics education?

Siemon (2006) argues that without developing Big Ideas, students' mathematical progress may be limited as Big Ideas provide organising structures that support further learning and generalisations. Writing about Big Ideas in the Australian curriculum, Siemon, Bleckly and Neal (2012) suggest that Big Ideas cannot be clearly defined, but can be observed in activity. However, I am of the opinion that we, the mathematics education research community, should be able to reach agreement on defining Big Ideas and then describe what these might be in practice.

Schifter & Fosnot (1993: 35) locate Big Ideas within the discipline of mathematics by defining them as "the central, organizing ideas of mathematics – principles that define mathematical order". Attempts to frame curricula around the Big Ideas of mathematics can be traced back to the 1960s, when, for example, the 'new mathematics' movement in the UK (and elsewhere) encouraged emphasis on set theory in school mathematics. Although mathematicians may have found set theory important in their work, the evidence is that it did not engage teachers or learners in the same way (Kline, 1974).

Similarly, locating Big Ideas primarily within the mathematics discipline and referring to them as "fundamental ideas in mathematics", Schweiger (2006) suggests ideas from different mathematical dimensions (history, content, difficulty and location) that include algorithm, characterisation, combining, designing, exhaustion/approximation, explaining, function, geometrisation, infinity, invariance, linearisation, locating, measuring, modelling, number/counting, optimality, playing, probability, and shaping.

While it helps to have examples, the variation among these ideas is problematic in terms of teaching implications. In what sense are verbs such as 'combining' or 'locating' Big Ideas, and how do they compare with mathematical objects such as 'algorithm'? How does one square off an idea such as 'invariance' with something equally broad but very natural such as 'playing'? I agree with Charles' observation that while "Big Ideas need to be big enough that it is relatively easy to articulate several related ideas, [… they also] need to be useful to teachers, curriculum developers, test developers, and to those responsible for developing state and district standards." (Charles, 2005: 11).

Charles (2005: 10) offers the definition that “A Big Idea is a statement of an idea that is central to the learning of mathematics, one that links numerous mathematical understandings into a coherent whole”, shifting the emphasis from the discipline of mathematics to encompass learning. He offers, as a starting point, 21 Big Mathematical Ideas for primary and middle-school mathematics, but again, the scale of ‘big’ varies, in the sense of how encompassing a Big Idea might be.

> *EQUIVALENCE: Any number, measure, numerical expression, algebraic expression, or equation can be represented in an infinite number of ways that have the same value.*
>
> *PROPERTIES: For a given set of numbers there are relationships that are always true, and these are the rules that govern arithmetic and algebra.*
>
> *ESTIMATION: Numerical calculations can be approximated by replacing numbers with other numbers that are close and easy to compute with mentally. Measurements can be approximated using known referents as the unit in the measurement process.*
>
> *VARIABLE: Mathematical situations and structures can be translated and represented abstractly using variables, expressions, and equations. (Charles, 2005: 12-18)*

Building on her work with Schifter, Fosnot (with Dolk) argues that Big Ideas also mark a shift in learners’ reasoning, with shifts in learners’ reasoning being analogous to the development of the discipline of mathematics, in that shifts in the history of mathematics are “characterized by paradigmatic shifts in reasoning”, just as learners’ reasoning also shifts through Big Ideas (Fosnot & Dolk, 2001: 11).

Likewise, Davis and Simmt (2006), taking a complexity stance with respect to learning, argue for broadening our view of learning by moving from treating learning as something that only happens in the heads of individuals, to regarding the discipline of mathematics as having learned over the course of its history. Through this wider view, both the individual and the discipline learning have much in common, although a key distinguishing feature is the time scale of the learning.

Towards a definition of Big Ideas

Drawing on this writing, I suggest the following criteria to frame whether or not something counts as a Big Idea in mathematics education. I agree that a Big Idea should be both culturally, that is mathematically, significant as well as individually and conceptually significant, as suggested by Fosnot and others. However, I also suggest that, in putting together a collection of Big Ideas, there are pragmatic considerations that address pedagogical issues over and above theoretical ones.

Pragmatically, an idea must be big enough to connect together seemingly disparate aspects of mathematics, but not so big that it is unwieldy. For example, ‘mathematics is about modelling’ is a Big Idea and it may be helpful in broad terms for framing the curriculum. Indeed, the Dutch Realistic Mathematics Education programme of research and curriculum development is built around this big idea (Gravemeijer, 1997), but it may be rather too big to be helpful in guiding pedagogy and focusing specific classroom actions.

On the other hand, an idea such as 'fractions, decimals and percentages all present different ways to represent a multiplicative relationship between two quantities' is a Big Idea that links together aspects of the curriculum often treated distinctly, and it is small enough to be thought about in terms of practical actions to bring such connections into being in classrooms.

A second pragmatic consideration is that the idea must have currency across all the years of primary schooling, because then children get to revisit Big Ideas across the year groups (Small, 2012). Over time, collectively and individually, the ideas will grow and develop, building on a core of similarity within and across the years. Thus, all children can be engaged in thinking about the Big Idea at different developmental levels. Working with Big Ideas becomes a means of dealing with classroom diversity and promoting inclusion.

Examples of Big Ideas

I now examine a small number of Big Ideas that might form part of a core collection for primary mathematics. I owe much to Charles' (2005) work in this instance, and while these Big Ideas are neither exhaustive of the number of Big Ideas that children need to meet, nor necessarily the most important of the Big Ideas, they are ideas that research and my experience suggest would improve learning if explicit attention were paid to them.

Big Idea: Place value. Our number system is built on ten digits, groups of ten and digits in specific places.

With only 26 letters in English, our alphabet allows us to express our ideas. Mathematics is more efficient – infinitely many numerical ideas can be expressed with only 10 digits – 0 to 9. Mathematically, place value is a Big Idea: the development of the history of mathematics would have been very different if no one had come up with the idea of place value as scaling up or down by factors of ten. It is also a Big Idea conceptually – it helps learners come to understand mathematics as a network of interconnected ideas, not a series of separate ones. For example, decimals come to be understood as an extension of the place value system through thinking of the values of digits being successively scaled down by factors of ten: ones scaled down to one tenth of their size, giving rise to the first decimal place, that, in turn, can be scaled down by a factor of ten, giving hundredths, and so on.

Awareness of place value as a Big Idea has the potential to alter how it is taught. Currently, place value is commonly treated as an additive process – exchanging 10 units for one ten, and 10 tens for one hundred (in preparation for the standard algorithms for multi-digit addition and subtraction). Were learners to meet the Big Idea of place value as a multiplicative scaling process based on the powers of ten, then later difficulties with ideas such as decimals and standard notation for numbers might be reduced.

Big Idea: Position on the number line. The numbers in the primary curriculum - counting, fractional and negative - all have a unique position on the number line. The idea that numbers are uniquely positioned is also conceptually important, as illustrated by these two problems:

> *A bar of chocolate is cut up into four equal pieces and Hamsa eats three of the pieces. How much of the chocolate does Hamsa get to eat?*
>
> *Hamsa and three friends share three bars of chocolate equally. How much of the chocolate does Hamsa get to eat?*

The answer in each case is three quarters of a bar, but as each arises from different situations, it may not be obvious to the learner that the quantity is the same in both instances. Modelling this on a number line shows that the answers are equal: the point on the line arrived at by marking three quarters of one unit turns out to be the same point reached by taking three units on the line and finding one quarter of that total length.

Conceptually, this Big Idea also introduces learners to notions of infinity: the counting numbers extend infinitely; there are an infinite number of fractions between any two numbers on the line and so forth.

Big Idea: Equivalence. There are infinitely many ways to represent numbers, measures and number sentences.

Mathematically, equivalence is a Big Idea because representations that seem different can all be linked to the same underlying idea. For example, 1/2, 50% and 0.5 are all equivalent representations of the idea of a half. Which representation we choose to use is linked to the context within which it arises; for example, fractions are often used in the solution to division problems with non-whole number answers; percentages to express probabilities, and decimals in the context of measures. But the underlying mathematics does not essentially change: two people equally sharing a pizza will get 1/2 each; my coin is likely to land on heads half the number of times I toss it; one metre of fabric cut into two equal pieces results in 0.5m.

Conceptually, equivalence is a Big Idea for several reasons. First, because it means that numbers and measures can be expressed in an infinite number of equivalent ways by different partitionings and factorisings, with different representations highlighting different properties. Take, for example, the number 64. We can express this as:

8 x 8
60 + 4
4^3
70 − 6
2 x 2 x 2 x 2 x 2 x 2
164 − 100
128 ÷ 2
100 – 36.

These different representations emphasise different things. From 2 x 2 x 2 x 2 x 2 x 2, we know that 3 is not a factor of 64, while 8 x 8 reveals that 64 is a perfect square and 4^3 that it is a perfect cube. Listing different equivalent expressions for 63 and 65 can illustrate just how different these are from 64, making all three more interesting than simply being three consecutive numbers in the sixties.

Equivalence is also conceptually important as when calculating it may be easier to answer an equivalent calculation than the one actually given. Figuring out, say, 49 x 4, one could reason that '40 times four is 160, nine times four is 36, so the answer is 160 plus 36, that's 196'. Alternatively '50 times four is two hundred, so the answer must be four less, that's 196'. As equivalent equations the reasoning goes:

49 x 4 = (50 − 1) x 4 = (50 x 4) - (1 x 4) = 200 − 4 = 196

Big Idea: Meanings and symbols. The same symbols or number sentences can be used to model different realistic situations, and different symbols or number sentences can be used to model the same realistic situation.

The Cockcroft Report's (1982) claim that mathematics provides an unambiguous means of communication echoes a popularly held belief that there is a simple one-to-one match between a realistic situation and the mathematical sentence that can be used to model that problem. Take, for example, 'My niece was born in January 1998. How old was she on her birthday in 2013?'

One mathematical sentence to go with this is 2013 − 1998 = [ ]. But this situation can also be modelled with 1998 + [ ] = 2013, or 2013 - [ ] = 1998 and, mentally, it is easier to say 'What do I add to 1998 to make 2013?' than it is to try and 'take away' 1998 from 2013.

In reverse, the same number sentence can be used to model different realistic problems. As the work of Carpenter and colleagues (1999) has shown, there are several distinct 'root' problems for each of addition and subtraction, and multiplication and division. The relationship between symbols and situations is not a simple one-to-one mapping, but a many-to-many mapping.

Mathematically, the many-to-many relationship between symbols and situations is a Big Idea, because it means a small number of symbols can be used to serve a whole host of problems. Conceptually, it is a Big Idea, because it means that thinking about how to represent a problem can affect the ease of arriving at an answer. Given a calculation, the sort of 'real-world' situation learners try and link with it can make a big difference. Anghileri (2000) demonstrated this when asking upper primary pupils what they thought would be the answer to 12 ÷ ½. Many tried to fit the symbols to a 'sharing' situation, but when they could not work out what a sensible meaning for 12 'shared' between a half, they interpreted the symbols as meaning 'what is half of twelve?'. Had they been able to fit the symbols to a division-as-repeated-subtraction situation – how many half pizzas can be served up from 12 pizzas? – then they might have arrived at the correct answer of 24 rather than 6.

Big Idea: There is a distinction between quantities and numerals.

Visiting a classroom many years ago, I overheard a conversation between a teacher and a pupil about carrying out a subtraction, say, 264 – 178, that went along the lines of:

> *T: Can we take eight from four?*
>
> *P: No.*
>
> *T: So what do we do?*
>
> *P: We cross out the six and put five and put a little one by the four.*
>
> *T: No, we borrow ten from the sixty to leave fifty and add it to the four to make fourteen.*

The teacher 'correcting' the learner suggests that the learner was 'wrong', when in fact she and the teacher were talking about different things. The pupil's explanation was at the level of what was literally done – those were the actions being carried out on the numerals. The teacher was talking about the actions that would be carried out on the actual quantities – which, in this particular instance, only existed in the teacher's head. Of course, once you have come to understand 264 as two 100s and 6 tens and 4 ones, it is easy to slip back and forth between quantity talk and numeral talk – but that ease takes some time to acquire. The distinction between quantities (amounts) and numerals (how we represent amounts) is one that teachers need to bear clearly in mind, else confusion can arise.

Big Ideas in CAPS

I now turn to where and how Big Ideas might be positioned in the CAPS FP and CAPS IP documents. It is beyond the scope of this article to examine a wide range of Big Ideas in these documents, so I focus on two: Equivalence and Meanings and symbols.

I chose these two Big Ideas, because they are slightly different in their orientation towards either the mathematical (discipline) or the conceptual (learner). Equivalence is a key Big Idea in mathematics as a discipline and any mathematics education not specifically attended to exploring and developing the Big Idea of equivalence would be lacking. Meanings and symbols, while mathematically important, is pedagogically important, as teaching for understanding this Big Idea would mean being alert both to how situations can be mathematically modelled in different ways and to how learners might be expected to express things mathematically in different ways rather than in one expected fashion.

The overall aims for the mathematics curriculum, overarching both the FP and the IP, include the aims that learners should develop, namely, appreciation for the beauty and elegance of mathematics, and deep conceptual understanding in order to make sense of mathematics (CAPS FP: 62).

Such aims can be read as in line with teaching for Big Ideas. The elegance of mathematics comes, in part, from the way in which Big Ideas link seeming disparate content, while, as argued earlier, deep conceptual understanding is linked to the growth of Big Ideas.

How does this play out in the details? The language of Big Ideas is not explicitly used in the CAPS documents. Therefore, I am seeking evidence of an acknowledgement of the importance of Big Ideas permeating the curriculum in the details of the documents.

Equivalence as a Big Idea in CAPS

The CAPS FP does not make explicit use of the term 'equivalence' at all. The word 'equivalent' is, however, used and related to three ideas:

- Equivalent fractions, for example;

  that 1 half and 2 quarters are equivalent (CAPS FP: 474).

- The introduction of multiplication as repeated addition;

  Repeated addition is often introduced to learners as groups of equivalent numbers. Initially learners can be introduced to everyday equivalent groupings (CAPS FP: 136).

- Equivalent representations;

  During this term the focus is showing equivalent representations for the same number. Twenty should be described as 2 tens (using the bundles or groups of objects) or 2 groups of tens. It is important to show learners that 20 can look different (CAPS FP: 217).

  Say which number is equivalent to or the same as:

  - *6 tens*
  - *Nine tens and three ones*
  - *Five tens and nine ones (CAPS FP: 13)*

The idea of 'equivalent representations' is close to Equivalence, but as these examples show (and there are only a small number of such examples in the document), Equivalence is restricted, in this instance, to place value; learners should notice 20 as two groups of ten, but there is no suggestion of other equivalences such as four groups of five or as 19 plus one.

Searching on 'equal' or 'equals' within the document brings up only references to the equals sign, for example, "can record their calculations using the plus (+), minus (-) and equals (=) sign" (CAPS FP: 340). There is no discussion or exemplification of the use of the equals sign to link equivalent representations.

The CAPS IP also refers to equivalence in relation to fractions, both in terms of equivalent fractions and different representations, for example:

- Equivalent forms:

  Recognize equivalence between common fraction and decimal fraction forms of the same number.

  Recognize equivalence between common fraction, decimal fraction and percentage forms of the same number (CAPS IP: 17).

This idea of 'Equivalent forms' also occurs with reference to numeric patterns and how these might be expressed in different ways:

- Determine equivalence of different descriptions of the same relationship or rule presented
  - *verbally*
  - *in a flow diagram*
  - *in a table*
  - *by a number sentence (CAPS IP: 19).*

There is increasing emphasis on equivalence in number sentences, although the messages concerning this are mixed. For example, in relation to Grade 4, the advice is that:

> *Number sentences are also a way of showing equivalence. It seems obvious that what is written on the one side of the equal sign is equal to what is written on the other side. However but (sic) learners need to be trained to understand the equivalence. In the Intermediate Phase it is useful to use number sentences as statements of equivalence (CAPS IP: 39).*

The idea that learners can be 'trained to understand' seems to be a contradiction in terms, and there is no explanation of what such 'training' might constitute. In the move to Grade 5, the advice includes:

> *As before, number sentences are used to develop the concept of equivalence. But they can also relate to all aspects of number work covered during the year. During the second part of the year you can give learners practice in answering multiple choice questions, which is a common format in national systemic tests (CAPS IP: 100).*

Here again, the messages are mixed. Rather than emphasising equivalence as a Big Idea in mathematics in its own right, the reader is left with the impression that equivalence may be important because of the form of national assessments. However, the examples provided do give a sense of equivalence in number sentences:

Which of the statements below is equivalent to 15 x (4 x 9)?

(a) (15 x 4) x 9

(b) 15 x 2 x 2 x 3 x 3

(c) (15 x 4) + (15 x 9)

(d) (10 – 1)(15 x 4) (CAPS IP: 205).

Which statement below is equivalent to: (26 x 39) + (26 x 1)?

a) 26 x 27

b) 400

c) 26 x 4

d) 26 x 40 (CAPS IP: 286).

In summary, there is some attention to equivalence as a Big Idea as the curriculum develops from Foundation Phase to the latter years of the Intermediate Phase, but there could be more consistency across the years. Unless a teacher brings awareness of this Big Idea, the curriculum document is unlikely to engage teachers and, consequently, learners with the Big Idea of Equivalence. Young learners can, for example, explore equivalences such as 7 + 8 + 3 = 7 + 3 + 8, leading not only to an informal understanding of equivalence, but also to using the equivalence to seek out effective and efficient calculation strategies.

Meanings and symbols as a Big Idea in CAPS

As argued earlier, the Big Idea that there is a many-to-many mapping between meanings and symbols is not only mathematically significant, but also conceptually important for learners to explore. Finding evidence for attention to such an idea in the curriculum documents is subtler than seeking references to equivalence, as there are less obvious key terms to seek. The approach taken was to look for examples and advice that might help teachers address this Big Idea in some form or another, particularly with respect to problem-solving and how different mathematical models might be set up.

In the CAPS Foundation Phase document, there are explicit references to problem types, the structure of these much in line with the typology of root problems developed by Carpenter, Fennema, Franke, Levi and Empson (1999). For example, across Grades 1 to 3, addition and subtraction problems are classified into change, combine and compare problems. In their analysis of such problems, Carpenter and colleagues emphasise how the position of the unknown in a problem can affect the level of difficulty, a point that is noted in the CAPS document. For example, the advice for Grade 3 includes:

Problems have to be posed in different ways. For example, both of these are change problems, but the "unknowns" are in different places in the problem:

> *The shop had packets of mealie meal and ordered 55 more. Now there are 170 packets of mealie meal. How many packets were there in the beginning?*
>
> *The shop had 500 packets of sugar. After selling some packets, they had 324 packets of sugar left.*
>
> *How many packets did they sell? (CAPS FP: 77).*

The guidance falls short, however, of the analysis provided by Carpenter and colleagues in two respects. First, their research shows that learners are likely to find the first of these problems – an example of a 'start unknown' problem – considerably harder than the second problem – a change unknown problem. Secondly, with respect to the Big Idea of meanings and symbols, there is no discussion of the different symbolic models that could be set up. For example, the first problem can be expressed as

[ ] + 55 = 170 or 170 − 55 = [ ] or 170 - [ ] = 55.

Which of these representations the learner chooses can alter the strategy used to carry out the calculation and the learner's likelihood of success.

At the end of the CAPS FP classification, the reader is advised to:

> *Note that learners often use different ways of solving a problem that may not be what the teacher expects. For example, a division problem may be solved by repeated subtraction, addition or multiplication. Learners' methods will change in the course of the year as their understanding of and familiarity with the problem types grow, and as their number concept develops (CAPS FP: 77).*

Whilst the spirit of this advice is laudable, the reader is left unclear as to whether or not different ways of solving a problem are to be welcomed, or whether, over time, the expectation is that learners will come to solve such problems in the way that the teacher does expect.

The CAPS IP provides similar examples of problem types. For instance, a grade 4 'calculate the change' problem given is:

> *A salesman earned R4 328 during November. During December, the amount increased to R7 435. How much more money did he earn during December than in November? (CAPS IP: 120).*

(Similar problems are provided for Grades 5 and 6, but involving larger numbers). Unlike the Foundation Phase document, no advice is given as to what to expect when learners engage with such problems, presumably on the assumption that setting up mathematical models for problems is not problematic in these Grades. This suggests, in turn, that these are not really problems to solve, but simply practice calculations wrapped up in words.

The CAPS IP comes closer to acknowledging the complexity of the relationship between meaning and symbols in its discussion of teaching fractions. For example, advice given includes the acknowledgement that:

> *Different diagrams or apparatus develop different ways of thinking about fractions:*
>
> *• Region or area models develop the concept of fractions as part of a whole. If used in particular ways they can also develop the concept of a fraction as a measure. …*
>
> *• Length or measurement models can be used to develop the concept of fractions as part of a whole and if used in particular ways also fraction as a measure. …*
>
> *• Set models develop the concept of a fraction of a collection of objects and can lay the basis for thinking about a fraction of a number e.g. 1/3 of 12.*
>
> *Learners should not only work with one kind of model, because this can limit their understanding of fractions (CAPS IP: 71-72).*

The strong message is that no one model or symbolic system for representing fractions is best. The relationship emphasised, however, is in the direction of establishing meaning through learners encountering a range of different symbolic representations. It falls short of examining the importance of the reverse relationship: that when given a fraction in symbolic form, choice of interpretation – region, length or set – can alter the ease with which the individual makes sense of the symbolic.

Again, the impression is that a reader with a sensibility towards Big Ideas will be able to find resonances within the document. But the reader less tuned-in to Big Ideas is unlikely to gain a sense of them when working with these documents.

Conclusion

I have argued that attending to Big Ideas in teaching and learning in primary schools can help learners develop a rich connected understanding of mathematics. On the basis of an albeit limited analysis of Big Ideas in the CAPS documents, I suggest that a lack of explicit attention to Big Ideas in the South African curriculum landscape could contribute to learners continuing to develop fragmented mathematical knowledge and, consequently, be a factor in the continued low attainment in mathematics.

Finally, I review some pedagogical considerations by returning to the work of Davis, Sumara and Luce-Kaplar and the idea that learning should not be thought of only as something that individuals do. They note that collectives can learn, raising another view on ‘Big Ideas’, namely that a collective may ‘have’ more sense of a Big Idea than any individual within a collective. As Davis and Simmt (2006) have demonstrated, teachers collectively exploring a topic such as ‘multiplication’ reveals a richness of understanding across a group that is more than the sum of the parts of the individual group members’ understanding. Big Ideas can, therefore, be worked with and developed through the collective bringing together of their individual understandings whereby “collectives of persons are capable of actions and understandings that transcend the capabilities of the individuals on their own”

(Davis, Sumara & Luce-Kaplar, 2000: 68). I suggest that this applies to learners as much as it does to teachers.

This view of Big Ideas as collective ideas presents a challenge to the prevalent discourse, in primary mathematics education, of having to differentiate teaching to meet individual needs, which often results in pedagogies that lead to a reduction in collective activity and group sense-making. A pedagogy built around the collective understanding and how the collective could build on, extend and develop Big Ideas brings a different view of classroom relationships and norms into being, one that

> *focuses on conversation patterns, relational dynamics, and collective characters. Cognition, in this frame, is always collective; embedded in, enabled by, and constrained by the social phenomenon of language, caught up in layers of history and tradition and confined by well-established boundaries of acceptability (Davis, Sumara & Luce-Kaplar, 2000: 65)*

Hence we can think of the Big Ideas of mathematics, as developed through history and tradition, as what mathematicians have defined and determined as acceptable. School learning is then constituted through dialogue and relationships and collective determining of what is acceptable, and meeting the needs of the individual learner comes about through that learner engaging with these ideas in dialogue and co-action with others. Big Ideas become Collectively Big.

References

Anghileri J 2000. *Teaching number sense.* London: Continuum.

Carpenter T, Fennema E, Franke ML, Levi L & Empson SB 1999. *Children's mathematics: Cognitively guided instruction.* Portsmouth, NH: Heinemann.

Charles RI 2005. Big ideas and understandings as the foundation for elementary and middle school mathematics. National Council of Supervisors of Mathematics (NCSM). *Journal of Mathematics Education Leadership,* 8(1): 9-24.

Cockcroft WH 1982. *Mathematics counts: Report of the Committee of Inquiry into the teaching of mathematics in schools*. London: Her Majesty's Stationery Office.

Davis B & Simmt E 2006. Mathematics-for-teaching: An ongoing investigation of the mathematics that teachers (need to) know. *Educational Studies in Mathematics*, 61: 293-319.

Davis B, Sumara D & Luce-Kaplar R 2000. *Engaging minds: Learning and teaching in a complex world.* Mahwah, NJ: Lawrence Erlbaum Associates.

Department for Basic Education (DBE) 2011a. *Curriculum and Assessment Policy Statement (CAPS): Foundation Phase Mathematics Grades R-3.* Pretoria: DBE.

Department for Basic Education (DBE) 2011b. *Curriculum and Assessment Policy Statement (CAPS) Intermediate Phase Grades 4-6.* Pretoria: DBE.

Fosnot CT & Dolk M 2001. *Young mathematicians at work: Constructing number sense, addition and subtraction.* Portsmouth, NH: Heinemann.

Gravemeijer K 1997. Instructional design for reform in mathematics education. In M Beishuizen, K Gravemeijer & ECDM Van Lieshout (eds), *The role of contexts and models in the development of mathematical strategies and procedures* (pp. 13-34). Utrecht: Freudenthal Institute.

Hiebert J & Carpenter TP 1992. Learning and teaching with understanding. In D. A. Grouws, (Ed.), *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 65-97). New York: Macmillan Publishing Co, Inc.

Kline M 1974. *Why Johnny can't add: The failure of the new math*. New York: Random House.

Schifter D & Fosnot CT 1993. *Reconstructing mathematics education: Stories of teachers meeting the challenge of reform.* New York: Teachers College Press.

Siemon D 2006. Assessment for Common Misunderstandings Materials. Prepared for and published electronically by the Victorian Department of Education and Early Childhood Development. Retrieved 26/04/13 from http://www.education.vic.gov.au/studentlearning/teachingresources/maths/common/default.htm.

Siemon D, Bleckly J & Neal D 2012. Working with the Big Ideas in number and the Australian curriculum: Mathematics. In B Atweh, M Goos, R Jorgenson & D Siemon (eds), *Engaging the Australian national curriculum: Mathematics – Perspectives from the field (pp. 19-45). Online Publication:* Mathematics Education Research Group of Australasia.

Small M 2012. Good questions: *Great ways to differentiate mathematics instruction. 2nd ed.* Reston, VA: National Council of Teachers of Mathematics.

Schweiger F 2006. Fundamental ideas: A bridge between mathematics and mathematical education. In J Maaß & W Schlöglmann (eds), *New mathematics educational research and practice (pp. 63-73)*. Rotterdam/Taipei: Sense Publishers.