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Petrusa du Toit, Dorothea Froneman & Kobus Maree

Mathematics learning in the foundation phase: facilitating a parent-teacher partnership

Summary

The approach to mathematical learning and teaching in South Africa has changed considerably in recent years. The new curriculum, Curriculum 2005, promotes a problem-centred, outcomes-based approach to mathematics instruction based on constructivist viewpoints. In this article, co-operation between parents and teachers involving the education of parents in the new approach to teaching mathematics, is advocated. Guidelines are provided for the encouragement and education of parents in the support of their children's mathematical learning. It is hoped that this will contribute towards stimulating parental involvement in the prevention of learners' mathematical problems, especially in the Foundation Phase, and ultimately towards better achievement in mathematics throughout education.

Wiskundeleer in die grondslagfase: fasilitering van 'n vennootskap tussen ouers en onderwysers

Die benadering tot die onderrig en leer van wiskunde in Suid-Afrika het die afgelope paar jaar drasties verander. Die nuwe kurrikulum, Kurrikulum 2005, poog om 'n probleemgesentreerde, uitkomsgebaseerde benadering tot leerfasilitering in wiskunde (gebaseer op die konstruktiwistiese epistemologie) te bewerkstellig. In hierdie artikel word samewerking tussen ouers en onderwysers sterk ondersteun. Dit beteken onder meer dat ouers ingelig word rakende die nuwe benadering tot die wiskunde-onderrig. Riglyne word voorsien aan die hand waarvan ouers aangemoedig en touwys gemaak behoort te word in die ondersteuning van hul kinders se wiskundeleer. Implementering van hierdie riglyne behoort 'n bydrae te lewer tot die fasilitering van ouerbetrokkenheid rakende die voorkoming van leerders se wiskundeprobleme, veral in die grondslagfase. Die einddoel bly egter beter wiskundeprestasie dwarsdeur die leerder se skoolloopbaan.

Prof P du Toit, Ms D Froneman & Prof J G Maree, Dept of Educational Psychology, Faculty of Education, University of Pretoria, Pretoria 0002; E-mail: nsopdt@puknet.puk.ac.za & jgmaree@hakuna.up.ac.za

Satisfactory co-operation between teachers and parents can play a major role in improving children's achievement in mathematics. Goldstein & Campbell (1991: 27) even maintain that

> the unpalatable fact which emerges from research on various kinds of help available to children who are experiencing difficulties is that the help from highly trained specialist teachers is less effective than that from children's parents.

They suggest that effective parental participation in their children's education may be one way of preventing specific learning difficulties from arising (Goldstein & Campbell 1993: 181). They also found that

> having parents work with their children to reinforce mathematics skills has been shown to enhance achievement levels in that subject for early elementary school pupils. Even mathematically unsophisticated parents can become natural partners of teachers in the educational process if given materials to use and guidelines for how to use them effectively and if also made to feel that they are making an important contribution to the child's academic progress (Goldstein & Campbell 1991: 27).

Stallings & Stipek (1986: 741) argue that "family involvement activities help foster positive attitudes towards school, and in turn support children to be successful in school and to be persistent enough to graduate". However, parents often do not know how to assist their child in extending classroom learning at home. The terminology used in mathematics, for instance, poses a barrier for parents. When parents approach the teacher for guidance, the work given often involves only practice in mechanical manipulations similar to the type of work done at school, so that understanding of the subject is not promoted. Sometimes the work given can be at too difficult a level for the learner to do without some explanation, which forces parents to do some teaching. At this point parents become confused because they are not familiar with the method of teaching used in the school, or the learner becomes confused because he is taught a different method at home from the one taught in school (Tregaskis 1991: 14). Some parents, even when their own learning of mathematics caused feelings of inadequacy and failure, expect their children to learn in the same manner they did (Stephens & Carss 1986: 12; Taylor 1992: 42). If this is not the case, parents fear that standards have deteriorated. This attitude can lead to over-high expectations of performance on tests, with consequent anxiety, dislike and distrust concerning mathematics on the part of children (Stephens & Carss 1986: 12).

Parental guidance in connection with mathematics is provided mainly by the teacher, the school or other helping professions (the psychologist, the remedial teacher or the occupational therapist). Parental guidance ought to enable and empower parents, as part of the everyday upbringing of their children, to prevent mathematical problems or to assist their children with mathematical problems. Parents often want information concerning their children's programme of work at school, the teaching methods currently being employed, or the new mathematics curriculum (Sawyer 1993: 195). It is therefore the task of the school and specifically of the teacher to provide opportunities for parents to obtain such information.

The approach to mathematical learning and teaching in South Africa has changed considerably in recent years. A new nationwide curriculum, Curriculum 2005, which promotes a problem-centred, outcomes-based approach to mathematics instruction (Department of Education 1997a) has been introduced in Grade 1 (learners aged up to seven years eleven months) since 1997.

This curriculum promotes learning through learners' own experiences, and a less formal style of teaching and assessment, with the focus on outcomes-based assessment. Curriculum 2005 was revised in 2000, but still adheres to the same constructivist and problem-centred viewpoints, with the emphasis on learners constructing their own strategies and solutions within mathematical problem-solving situations. The pedagogy promoted by Curriculum 2005 differs from the ways in which parents learned mathematics, encouraging co-operation between parents and teachers and stimulating parental involvement, which requires that not only parents be encouraged to help their children, but also that they be educated in the new approach to mathematics teaching. By involving the parents, an extra resource is created in the learning process. Parents can stimulate and promote their children's interest in mathematics. Stanic (1989: 34) advocates early intervention programmes, particularly to ensure that children from single-parent homes, poor children, or children from disadvantaged or non-English-speaking backgrounds start school with an even chance and do not fall behind. He claims that the most successful of these programmes make use of parental involvement. Parental involvement in learners' progress is officially recognised as crucial (Department of Education 1997b: 18, 38, 102):

> Where parents' participation is not facilitated and encouraged, effective learning is threatened and hindered. Negative attitudes towards parental involvement, lack of resources to facilitate such involvement, lack of parent empowerment, particularly in poorer communities, all constitute barriers to learners' mathematical learning [...] policies have yet to be transformed into structured parent development and empowerment strategies and programmes.

1. Aims of the study

The aim of this study is to indicate how closer co-operation between parents and teachers can be facilitated, by providing guidelines to teachers and parents on the constructivist and problem-centred approach to the teaching and learning of mathematics. To understand and prevent mathematical problems in their children, these problems and their causes should be clear to parents. This information should be made available to parents in the form of guidelines on how to help their children with mathematical problems as well as how to prevent mathematical problems by means of enrichment of their children's mathematical learning.

2. Clarification of terms

Potential guidelines for teachers and parents for the prevention of mathematical problems will now be presented, based on constructivist and problem-centered approaches in the teaching and learning of mathematics. In the following discussion a number of terms which reflect and explain the consequences of choices regarding particular paradigms in both mathematics and teaching will be used, which may need prior clarification.

2.1 Mathematics

Mathematics can be defined as the study of numbers, measurements and space. It is a science dealing with measurement, properties, and relationships of quantities, as expressed in numbers or symbols. Rothman & Cohen (1989: 133) state that "mathematics may be regarded as a symbolic language whose practical function is to express quantitative and spatial relationships".

Calculations or arithmetic, on the other hand, involve abilities like counting, used to solve mathematical problems. Moreover, cognitive skills like comparing and analysing serve to enhance the solving of mathematical problems.

2.2 Mathematical problems and mathematical errors

Problems in mathematics are experienced by a learner as an inability to solve a mathematical problem correctly, which may manifest in mathematical errors, for example not understanding which calculation process to use and then using the wrong calculation process.

2.3 Foundation phase

The foundation phase in South African education encompasses all learners in Grades 1 to 3, irrespective of age (ages vary from six years to nine years eleven months). According to the South African Schools Act (Government Gazette No 17579 1996: 7) learners start school (Grade 1) in the year that they turn seven.

2.4 Mathematics teaching and learning

This article is in agreement with the views of Grossnickle *et al* (1983: 8-10), who state that:

- The teacher is the key to change and innovation in learning mathematics. This means that teachers should not only have mastered the content of their subject, but should also understand children and the way they learn and understand mathematics. They should be aware of underlying psychological principles when they determine a suitable level of learning for children at various stages of their mental development.
- Problem-solving strategies should be accorded priority in any syllabus for mathematics.
- Clearly defined goals and objectives form the basis of a comprehensive, balanced approach to learning and teaching mathematics. This means that children should, among other things, develop the following:

- the ability to think quantitatively in problem-solving situations,
- a functional knowledge of the language and structure of mathematics, including the ability to estimate, to approximate and to judge the feasibility of the results of problem-solving situations,
- sensitivity to a wide variety of quantitative situations in real life and the ability to apply mathematics to everyday situations,
- an intelligent mastery of arithmetical skills and abilities including insight into the reasons why certain mechanical computations are necessary,
- an appreciation of the use and importance of mathematics in modern life, and
- a sound, positive attitude towards mathematics and the opportunities it creates to learn and discover.

2.5 The new approach to teaching and learning mathematics in South Africa

It is often alleged, rather unscientifically, that mathematics is changing in schools. This, of course, is not true. It is the approach to teaching and learning mathematics at school level that is changing — not mathematics itself. Although people refer to the "new maths", they actually mean the problem-centred approach to teaching and learning mathematics. The main theory underlying this approach is known as (social) constructivism. According to this approach, knowledge is acquired, and cannot be given or transferred. Apart from the fact that acquiring the ability to solve problems is a good reason to study mathematics, it also provides a context in which mathematics can be learned and practised. The focus thus shifts

- from the child as someone who can do something, to the child as someone who can think actively;
- from concepts and skills, to concepts, skills and processes;
- from mastering algorithmic skills, to developing algorithmic thinking, and

- from the application of mathematics to solving problems, to problem-solving as a method of investigation (Adler 1992: 28).

This approach emphasises, *inter alia*, the importance of social interaction, working together in groups, problem-solving, an enquiring mind and the involvement of learners in classroom activities (Volmink 1993). The discovery or creation of new mathematics is not seen merely as a logical, deductive activity. Discussion is an inevitable component of learning, as are the negotiation of meanings, quasi-empirical criticism and testing, logical argument and opportunities to develop independently in the construction of new mathematics.

3. The constructivist model as a theoretical basis for the problem-centred approach in the teaching and learning of mathematics

Du Toit & Wessels (1991: 66) define constructivism as follows:

> For the pupils this means active learning that builds on experience. For the teachers involved this means standing back and not teaching, telling or showing but rather facilitating developmental learning by supporting and challenging the pupils. In the process, pupils develop their own computational strategies which they understand because they are self-generated.

Olivier *et al* (1990: 364) state that:

> [C]onceptual knowledge cannot be transferred ready-made from one person to another, but must be actively built up by all children on the basis of their own experience. The teacher therefore becomes less a dispenser of knowledge and more a facilitator of learning, supporting and guiding children to construct their own knowledge. Children's ideas are respected and valued and the child is seen as an active participant in the learning situation, not a passive receiver of knowledge.

The constructivist approach to the learning and teaching of mathematics differs considerably from the traditional approach (Langford 1989: 150; Maree 1995: 68; Olivier 1989: 11), in the following ways:

- learners are seen as active participants and not as passive recipients of knowledge;

- learners create new mathematical knowledge by reflecting on their physical and mental actions and not only by absorbing knowledge from the environment through their senses;
- knowledge cannot be transferred ready-made and intact from teacher to learner, as was claimed in the traditional approach;
- learning occurs when a new concept is incorporated into learners' existing schema (a unit of interrelated ideas in their mind) and that schema is changed, not as a matter of adding (or stockpiling) new concepts to existing ones;
- misconceptions are important to the learning of mathematics, in that they help learners (and teachers) gain insight into their erratic thinking processes, and
- the emphasis is on learners' understanding the rationale for calculation procedures in the Foundation Phase, and not only on the correct implementation of those procedures and skills.

Teachers have to get learners actively involved in finding their own calculation procedures, especially in the light of the increasing number of learners in classrooms in South Africa. The constructivist approach to the teaching of mathematics has probably paved the way for the implementation of the Curriculum 2005 policy (Department of Education 1997a) in South Africa since 1997. Specific mathematical outcomes have to be mastered by all learners by the end of the foundation phase, *inter alia* estimation as a skill, performance of the four basic operations (addition, subtraction, multiplication and division) and solving of mathematical problems via the testing of hypotheses. The constructivist theory focuses on learners' own construction of mathematical knowledge through a problem-centred learning approach, which is now nationally accepted for use in the learning and teaching of mathematics. This approach will be discussed in the following section.

In the problem-centred approach, learning is facilitated by providing opportunities for learners to communicate and negotiate with other learners in the class about problem solving and then to restructure their own meaning by incorporating the ideas of others (Lo *et al* 1994: 46; Wheatley 1992: 529). Human *et al* (1993: 1) explain that the learners' task in this approach is to solve problems independently, using their own strategies. They must be able to explain their strategy to one another, assess it and compare it with the strategies of others. This will give them the opportunity to discuss, criticise, explain and, most importantly, justify their interpretations and solutions. Wheatley (1992: 529) claims: "Problem-centred learning is an instructional strategy based on constructivism". He further maintains that it is assumed that learners will give meaning to their experiences in idiosyncratic ways and that attempts to impose mathematical procedures are ineffective. The teacher rather has to present the tasks to be done with a minimum of instruction and allow learners to solve them independently, using their own strategies. A discussion about solving the problems then follows. The teacher's role is to facilitate the discussion, with the purpose of stimulating and activating learners so that, as far as possible, they achieve the various outcomes, such as reasoning logically, evaluating and justifying their own hypotheses and seeking the most effective method, through their own initiative. The teacher should allow apparently wrong contributions to a discussion in the light of their potential fruitfulness for further mathematical learning. Teachers must assume that learners' mathematical actions are reasonable from their own perspective, even if that perspective is not immediately apparent. For this reason, they are not allowed to communicate any negative judgement of learners' actions or thinking. Teachers do not mark worksheets wrong, or have learners correct their "mistakes". Instead, they keep notes on learners' persistence, confidence, co-operation and communication, as well as the quality of their mathematical computation, all of which contribute towards their comprehension in mathematics. The fewer rules the teacher prescribes, the better the learners understand their own mathematical reasoning. Verbal interaction is of the utmost importance, with learners being given the opportunity to explain the procedures used in arriving at the answer, because this helps them to structure and order their reasoning.

1 Cf Brissenden 1989: 26; Cobb *et al* 1992: 486; Human *et al* 1993: 1; Wheatley 1992: 23.

2 Cf Bednarz *et al* 1993: 48; Human *et al* 1993: Maree 1994: 41; Maree 1995: 69.

Although this approach is aimed at minimising mathematical problems, problems will still occur and need to be taken note of by teachers and parents, in order to try and prevent their recurrence. Teachers need to help parents understand the nature of the mathematical problems experienced by their children, as well as the factors contributing to these problems, to ensure better parental support for their children. Gannon & Ginsburg (1985: 409) comment that

> Piaget pointed out that it is necessary to look beneath the child's errors in order to discover underlying patterns of thought or other factors that cause them [...] Failure is a symptom, not a disease. And as with any other symptom, failure in school mathematics can have several distinct causes.

Some of the problems experienced in the traditional approach are still evident in the constructivist model, irrespective of whether they are due to poor teaching practices or to factors within the learner. These include the following:

- Learners experience problems with estimation of quantities and qualities such as distance, volume, mass, time, width, size and length (Grové 1993: 251-2).
- Learners struggle to make a sum of a story, for example: "Paul has four sweets. He eats three. How many has he got left?" (Grové 1993: 252).
- Learners struggle with the language of mathematics. The terms "plus", "minus", "subtract", "multiply", etc may be unfamiliar to them and confuse them during computation (Ginsburg 1977: 143).
- Interference, which implies a wrong choice of (more familiar) methods when confronted with a new problem (ie when a method learnt earlier interferes). For example, when the question is asked: "How many cookies are there in four jars with four cookies in each of them?" (four times four), a learner may incorrectly answer "eight" (four plus four — addition having been learnt earlier than multiplication). "When a second question is asked: "What is four plus four?", the learner may then correct his/her previous answer when s/he realises the mistake, but may make another mistake on the second question by answering "sixteen". These learners understand multiplication, but instinctively retrieve the addition schema (the knowledge structure learnt first).

- Learners sometimes change the instructions of a mathematical assignment, when they perceive it as too difficult. They knowingly or unknowingly read or interpret the instructions wrongly.
- Learners struggle to understand different formats presenting the same problem, because to them the meaning differs. Olivier (1989: 16) predicted that these errors would persist in the constructivist model. He cites an example to explain this phenomenon:

  $$5 \times 3 = 3+3+3+3+3 \text{ and } 3 \times 5 = 5+5+5$$

  The format looks different and thus tends to have a different meaning for learners.
- Overgeneralisation of the commutative property (in which the result is the same although the order or sequence changes) also occurs. The rule states that in addition and multiplication commutation can be applied. Some children overgeneralise this rule and wrongly believe that subtraction and division are also commutative. To these learners 6-4 and 4-6 are the same or have the same answer (Davis 1984: 115; Olivier 1989: 15).
- Improper repeated subtraction frequently appears during written work. Human, Murray and Olivier (1993: 80) cite an example:

  $$53 - 28 \rightarrow 50 - 20 \rightarrow 30 - 8 \rightarrow 22 - 3 = 19$$

  In this example the learner repeatedly subtracts all the numbers from each other, whereas s/he was supposed to add the 3 to the 22, not subtract it.

Various causes contribute to these mathematical errors. It is necessary to identify the causes in the incorrect learning of mathematics so that parents and teachers can be sensitised to them, in order to prevent them and be better able to assist learners. Within the limited space of this study and so that there can be more focus on the guidelines for parents, only the most important causes are briefly indicated here:

- learners' lack of appropriate strategies for solving mathematical problems;
- learners' poor concepts of direction and time;
- learners' memory problems;
- learners' anxiety, low self-concept and poor motivation, and

- parents' improper attitude towards mathematics.

4. Methodology

Qualitative research in terms of a literature study was undertaken on the constructivist, problem-centred model of learning and teaching mathematics, as well as on errors in learning mathematics and the causes of these. An empirical study was not undertaken, mainly because the recency of the model as prescribed by Curriculum 2005 and Curriculum 21 might have meant that teachers would have responded with uncertainty about the expectations of such an empirical study, which might have made it less scientifically verifiable.

5. Limitations of the research

It should be borne in mind that the qualitative research method has potential limitations and shortcomings and it is always possible for other researchers to interpret the material differently. This fact limits the potential for inference. The guidelines provided here in terms of intervention strategies to be implemented by parents can only provide evidence for treatment plans in specific cases, and do not prove anything. It is also accepted that, to strengthen the conclusion that the intervention strategies will be effective, they should be applied in a number of preferably heterogeneous cases. While the authors' observations were made in a relatively uncontrolled context, which may lead to erroneous conclusions, it is nonetheless assumed that much can be learned from the current research.

6. Guidelines for teachers and parents

Mechanisms by means of which encouragement of learners by their parents, as well as the teaching of mathematics by their teacher, should take place, include the following:

6.1 Facilitating acquisition of the limited, technical language of mathematics

Mathematics has a limited, technical vocabulary of its own, which needs to be taught/learned (Piaget 1971: 44; Rothman & Cohen, 1989: 141; Sharma 1981: 61-71), but nowhere in our current curriculum is provision made for instruction in the language of mathematics. Yet, arithmetical skill, and above all the ability to solve problems, is determined by the extent to which the learner has mastered the language of mathematics.

In order to master the entire vocabulary of mathematics at primary school level, a learner has to be able to understand and use about 350 to 400 mathematical words, as facilitated by parents and teachers. These mathematical words range from simple concepts such as "arrange" and "search" to more specialised concepts such as "triangle" and "quadratic".

It is essential for learners to master this mathematical vocabulary and to ensure that they know how to use it correctly (Sharma 1979: 5-22). In this regard, Sharma (1981: 61-71) points out that the learning of mathematical vocabulary assumes an interaction between three centres of mathematical vocabulary, namely

- that of the learner,
- that of the teacher, and
- that of the textbook.

The ways in which teachers and textbooks use mathematical vocabulary and formulate questions are often altogether different, leaving the learner trapped by concepts and formulations that are probably confusing him/her totally. While some learners have a more natural ability to decipher the different forms of expression, just as some have a natural knack for spelling, others are less capable of doing so. In some instances the learner who has a language problem finds it equally difficult to understand mathematical vocabulary and questions.

6.2 Handling problems relating to notation

It is essential to mention the problems experienced with regard to the different notational forms of calculations. The fact that problems occur is not surprising if one considers, for instance, that although $3.\frac{1}{2}$, $3(\frac{1}{2})$ and $3 \times \frac{1}{2}$ all mean the same, the term $3\frac{1}{2}$ is often confused with them, because it has the same surface structure. If a learner struggles to understand the different meanings of the symbols in the different notational forms of multiplication and division, s/he will find it difficult to understand why $3\frac{1}{2}$ cannot be $1\frac{1}{2}$ — the $3\frac{1}{2}$ having been incorrectly interpreted as $3 \times \frac{1}{2}$. All words, notations and symbols that may lead to confusion in calculations should be given special attention by the teacher during every mathematics lesson in calculation. Table 1 contains some examples of such terms and notations (Rothman & Cohen 1989: 139).

Table 1: Symbols, words or notations which may be confused

Symbol, word or notation	Correct description	Confusion
x	Times	x or incorrect
–	Minus	punctuation mark
( )	Bracket	punctuation mark
,	Decimal comma	punctuation mark
1/10	One tenth	tenth (ordinal number)
Even numbers	2; 4; 6	linguistic meaning
$2\frac{1}{2}$	2.5	$2 \times \frac{1}{2}$

6.3 Helping learners to enlarge their vocabulary meaningfully

The natural environment offers the pre-school learner numerous opportunities to develop a feeling for addition and subtraction. When young children finish their food, their parents dish up “some more”; they play with a toy and a brother/sister will “probably” take it away from them; when playing together they have to share or “divide” the toys, etc.

Between the ages of two and five learners develop many informal mathematical concepts and skills. During this time, discussions between parent and child, between teacher and child, and among children themselves, should ideally take place. Discussion is a very powerful form of communication and the learning of mathematics is greatly enhanced when discussion is optimised.

6.4 Helping young learners to communicate mathematically

The learner's language development is determined to a large extent by his/her experiences. In the same way mathematical-linguistic development should be accompanied by concrete experiences — learners need to be physically active to truly experience and understand concepts of volume, space, distance, weight, grouping, time, size, sequence, etc. Through these activities the foundation for mathematics is being laid. Parents ought to be made aware of this necessity for the development of mathematical learning in their children's lives, to ensure that they provide sufficient opportunities for their children's physical experiences, while also discussing these experiences with them. To successfully communicate concepts of volume, time and size, learners should be taught by their parents how to use the words in Table 2 (Rothman & Cohen 1989: 133).

Table 2: Words which may be taught to describe the concepts volume, time and size

Volume		Time		Size
Full	More	Before	When the bell rang	Small
Many	Fewer	After	Last summer	Little
Empty	Less	Yesterday	Spring	Big
A lot		Now	Winter	Tall
		Later	When it gets warmer	Short

6.5 Helping learners to understand the relationship between the understanding of language and word sums

The relationship between the understanding of language and problem-solving can be illustrated in the following word sum: Due to a shortage of wheat the price of bread was increased by 2 cents. When supplies became freely available again, the price was decreased by 4 cents. Later on, due to inflation, it was increased again by 5 cents. If bread originally sold at 220 cents a loaf, what was the price after the last increase?

To find the solution to this word problem, the learner needs to know the meanings of the maths-related words in the sum: "shortage", "increase", "decrease" and "inflation". S/he has to decipher the language and be able to reason about the word problem, by way of language, before s/he will be able to quantify it to a mathematical equation. Parents and teachers have the responsibility to help learners to define the mathematical words, symbols and language in word sums clearly and correctly (MacGregor 1986: 10; Nicholson 1990: 71). Dressler & Keenan (Rothman & Cohen 1989: 136-9) suggest that the words in Table 3 be explained and demonstrated by parents and teachers as alternatives to the formal concepts of the four basic operations.

Table 3: Alternative words to describe the four basic operations

Addition	Subtraction	Multiplication	Division
Plus Is added Increases by The sum of More than Exceeds	Minus The difference between Subtracted from Decreases by Becomes smaller by Less than Reduced by	Times The product of Multiplied by	Divides by Grouped into

6.6 Promoting problem-solving

First, the parent must let the learner feel that mathematics problems are real and interesting enough to be worth solving, and must allow plenty of time for this (Atkinson 1992: 28-29; Rocher 1988: 29). Other guidelines for facilitating problem-solving by learners include the following:

- The parent must allow the learner to search, discover and form conclusions independently (Steyn *et al* 1988: 40).
- Learners should be allowed to use various strategies or methods as well as different representations (number lines, graphs, diagrams, and drawings). Strategies such as searching for number sequences (5, 10, 15, 20…), always starting with the bigger number when computing, and breaking up a problem into smaller, simpler units should be taught and practised. Teachers and parents should not force learners to use only one "efficient" method, or show learners the method they consider works best, or suggest that there is "only one answer" and "one correct method". This breeds the

myth that mathematics is a magical, difficult or rigid subject.[3]

- The learner should be encouraged to talk during mathematical problem-solving activity, like thinking aloud, making predictions out loud, asking questions, explaining and discussing concepts with parents and teachers.[4] The following questions can be asked by parents to encourage reflection (Wheatley 1992: 535):

  "Please explain what you were doing."

  "Why will that work?"

  "How did you do that?"
- Although a parent should encourage perseverance, there should be no pressure to complete a problem which seems too difficult, because it may induce anxiety (Hawkey 1987: 47; Human *et al* 1993: 3)
- Wrong answers should not be dismissed without investigation. Often learners are giving the "right" answer to the question as they perceive it (Hawkey 1987: 28). Furthermore, learners should be encouraged to ask questions when they have a desire to know more about a mathematical procedure, and asking questions in itself indicates a relationship of trust between learners and their parents (Steyn *et al* 1988: 51). Parents' answers to questions should therefore always be prompt, lest the attention of the learners wander, their interest dwindle, and the trusting anticipation of an answer vanish. This is especially true in the Foundation Phase where the attention span of a learner is still very limited. It is not expedient to answer learners' questions with the evident answers, but rather to try and lead them into finding answers for themselves (Steyn *et al* 1988: 52). When learners have to work out the question for themselves, they take responsibility for solving the problem (Wood *et al* 1991: 608).

3 Cf Burton 1984: 9; Denvir *et al* 1982: 49; Flexer & Topping 1988: 19; Hawkey 1987: 28; Saarimaki 1993: 506; Sawada & Nelson 1994: 23; Young & Maulding 1994: 37.

4 Cf Brissenden 1989: 212; Bruneau 1988: 17; Ford & Crew 1991: 12; Ginsburg 1977: 172.

- Games with word problems containing direction, for example a treasure hunt, can be played with learners who struggle with the concept of direction, where directions are given, such as: "Let's search for the Easter eggs that are buried in the garden. Start outside at the front door. Walk six paces to the left and then ten paces towards the street".
- Predicting or planning ahead: The parent can ask the child to predict the outcome of events. For example, after he has built a tower of blocks, he has to predict what might happen if three more blocks were to be put on top. Or "what would you do if the blocks kept falling off?"
- When learners proclaim that they cannot do a problem, the parents should emphasise that they are only "stuck". This ought to be a temporary state and learners should be encouraged to "unstick" themselves without parental intervention. This gives them independence and improves their confidence and self-esteem. When they are stuck, learners should be encouraged to say what they need to know and what they already know. This makes being "stuck" part of the normal experience of mathematics.
- Human *et al* (1993: 9) advise that parents can use cognitive conflict, where learners are confronted with a mistaken overgeneralisation. For example, a learner overgeneralises the rule to subtract the "smaller number from the larger number" and then does an incorrect computation, eg:

  $$93 - 78 \quad 90 - 70 = 20$$

  A cognitive conflict situation can be created within the learner by then asking him/her to shift the position of the bigger and smaller one-units, ie to subtract 98 from 73, where the 8 (which is bigger than 3) is combined with the biggest unit-ten number (namely 90) and the smaller number (namely 3) is combined with the 70. The learner will obtain the same answer (25), and the question can then be asked whether it is possible that these two questions can have the same answer. S/he is then encouraged to look for his/her error in the mistaken overgeneralisation. Repeated subtraction problems can be handled in the same way.

6.7 Promoting estimation

Rocher (1988: 29) maintains: "Only pupils who can estimate understand." Promoting estimation can encourage intelligent guessing and intuitive responses. Flexer & Topping (1988: 17) encourage parents to make use of guessing activities, for example, estimation in money matters can be applied at the restaurant when the bill arrives: family members can first guess what the total is. The parents need only respond with "It's more" or "It's less". Learners should also be encouraged to make their own real purchases and handle their own real money when shopping with the parents, with little supervision.

6.8 Combating anxiety

Parents should make sure that their child experiences a degree of success in performing mathematical activities at home. As many positive mathematical experiences as possible should be provided (experiences that are relaxed, enjoyable and at the learner's level of development). This will lessen anxiety and also develop the learner's willingness to attempt problems, for learners often avoid attempting problems in order to protect themselves from defeat or humiliation. Parents should also be careful not to make condescending remarks to their children about their mathematical performance, for example "Can't you think properly?" or "You should know that!"

6.9 Enhancing self-concept

Hamachek (1975: 61, 308) maintains that learners' sense of competence and self-esteem is strongly affected by their experience in the family circle. Parents who succeed in enhancing their child's sense of competence and self-esteem, are usually those who have provided the means for achieving success (for example by making sure that a calculator or objects for counting with are always available and at hand). Baroody (1989: 12) advises parents to purposefully apply mathematics to everyday activities in which mathematics already plays an integral part, for example cooking, because it entails counting, measuring and fractions. While doing these activities, learners may not even realise that they are practising mathematics. Bruneau (1988: 17) also mentions that parents should be on the look-out for mathematical applications in the environment (for example have a child read numbers on license plates of passing vehicles, or find the price on a grocery item in a store, or name a coin, or count items on the table, or read the sports scores on television, or locate and identify various shapes in the vicinity). Daily mathematical activities like shopping, compiling the family budget and estimating how much food is needed for a meal can all be shared with the child. In these activities parents should not focus on sums, but rather try to broaden their children's comprehension of mathematical applications in everyday life, and boost their confidence therein.

6.10 Enhancing motivation

Pollard (1990: 69) remarks:

> Children learning at the limits of their understanding and skill pose us a particular challenge, for they are vulnerable both intellectually and motivationally. There is a real risk of misunderstanding and of demotivation. Our task is to recognise these crucial moments and to support the children in ways that leave them in control and ownership of the learning process.

Stander (1991: 214) expresses the view that motivation relates strongly to a positive self-concept. The following are important for parents to remember concerning the nurturing of motivation in their children (Stander 1991: 214; Grossnickle *et al* 1983: 17, 350):

- appreciation and encouragement for the learners' work (for example using positive reinforcement, praising them for good work done and encouraging them when they struggle) will encourage them, and
- relating the learners' learning to its usefulness, by creating problems that are part of the environment.

6.11 Improving awareness of parents' attitudes and expectations

Apart from promoting the above aspects in their children, Brissenden (1989: 210) maintains that parents' attitudes to mathematics could have a greater influence on their children's achievement than their actual school marks. Parents' attitudes thus play an important role in motivating and encouraging their children towards better mathematical performance. Parents should be sensitive to the following ways in which they may relay their attitudes towards mathematics to their children:

- They should avoid saying that they themselves were not good in mathematics, and rather trust that their children do have the ability to do mathematics (Atkinson 1992: 164; Saarimaki 1993: 506)
- Parents should be patient. They should not expect everything to be fast and easy — problems take time to solve (Saarimaki 1993: 506)
- Parents should be flexible. Many ways can be used to solve problems. They should try not to tell the answer but rather to discuss the problem and listen to the child's explanation (Saarimaki 1993: 506).
- Parents should hide their own mathematics anxiety, bearing in mind how easily attitudes are passed on to their children (Bruneau 1988: 17).
- Parents should believe in their children's abilities and expect their best possible efforts, but not necessarily good marks (Jacobs 1991: 526).

6.12 Utilising resources

Parents can use various resources to enhance the mathematical learning of their children. In this section computers, calculators, games and literature will be discussed.

- *Computers*

The personal computer affords excellent practice in problem-solving, in that the programme asks the learner a question and provides clues if s/he has difficulty. The computer continues to provide information until the learner succeeds. When s/he arrives at the correct answer the computer congratulates him/her. Computer programmes can also enhance concepts formation, devising and revising techniques, applying ideas, developing problem-solving skills, diagnosing mistakes and correcting misunderstandings. The computer is also usually the only "one" that "knows" that the learner was wrong and it is not dismayed by frequent mistakes (Anderson 1982: 369). Most computer programmes provide a structured learning environment without directly involving teachers or parents, but do provide them with opportunities to help learners understand mathematical concepts, devise problem-solving ideas, and apply these ideas in real problem-solving situations.

- *Calculators*

Drosdeck (1995: 305) maintains that learners will only be able to decide for themselves when the use of a calculator is appropriate after being given enough practice with the calculator in various assignments both at home and at school throughout the year. He expresses the view that calculators can easily be integrated into estimation activities. Learners should be encouraged to practise various estimation strategies by mentally computing the numbers in question and comparing their estimate with the actual answer on the calculator.

- *Games*

Raban & Postlethwaite (1988: 15) believe that an important part of any game should be discussion between parents and their child about the problem-solving strategies that the child may use during the game. This helps the child to actively investigate the possibilities of the game. S/he can also be reminded of these strategies in real-life situations. Bruneau (1988: 17) advises parents to use short, enjoyable mathematical games which will ensure success for the child. Moreover, games provide an opportunity to learn important social skills, such as how to deal with competitiveness in mathematics, so that the learner can learn that it is acceptable to lose in a game. Denvir *et al* (1982: 85) confirm that "the use of appropriate games is of particular value because they can be highly motivating and provide an opportunity for pupils to practise and consolidate their (mathematical) knowledge". If games are to be effective, the following principles should be taken into consideration by parents in support of their children's mathematical learning (Denvir *et al* 1989: 85):

- rules must be simple, unambiguous and fair;
- the game should result in a clear winner;
- the game should have an element of luck built into it, otherwise the most skilful player will always win, which could be the parent;
- games should encourage learners to verbalise their methods;

- it is helpful if the game allows time for the learners to correct themselves, and
- games should preferably be relatively short so that several repetitions are possible in the available time, in order for skills like problem-solving to be practised.

- *Literature*

Some books have worksheets with space for the learner to do calculations. Whitin (1994: 5) says: "Using math-related children's literature can help children realise a variety of situations in which people use mathematics for real purposes". Atkinson (1992: 164-5) elucidates that most picture books have considerable mathematical content, for example in counting activities and in words indicating position, measuring and quantity, for example "between", "through", "under", "around", "heavy", "more than", "enough", "smaller", and "late", as well as words indicating probability, such as "probably", "might", "chance" and "possible". Sequencing of events in stories is also a vital skill for the understanding of steps in mathematical problem-solving, because learners learn through stories that events take place in chronological order, and that steps in mathematical problem-solving must also be executed in chronological order.

Curcio *et al* (1995: 370) as well as Young & Maulding (1994: 37) note that humorous poems can also be used because poetry can spark a great deal of talk about mathematics and can involve estimation, as well as devising and comparing problem-solving strategies, for example:

"What can I do?
This library book is 42 years overdue.
I admit that it's mine
But I can't pay the fine. [...]"

7. Conclusion

The approach to mathematical learning and teaching in South Africa has changed considerably in recent years. A new nationwide curriculum, Curriculum 2005, which promotes a problem-centred, outcomes-based approach to mathematics instruction (based on constructivist viewpoints) was introduced in Grade 1 in 1997, promoting learning through learners' own experiences and a less formal style of teaching and assessment, with the focus on outcomes-based assessment. The pedagogy promoted by Curriculum 2005 encourages co-operation between parents and teachers and stimulates parental involvement, which requires that parents be educated in the new approach to teaching mathematics. Satisfactory co-operation between teachers and parents can play a major role in improving children's achievement in mathematics.

Mechanisms in respect of which education by teachers should take place, include facilitating learners' acquisition of the limited, technical language of mathematics. Furthermore, the learner must feel that mathematics problems are real and interesting enough to be worth solving, and plenty of time should be given for solving each problem.

It is hoped that this article will contribute towards parental involvement in the prevention of mathematical problems among learners, especially in the foundation phase, and ultimately towards better achievement in mathematics throughout education.

Suggestions for future research include the following: specific kinds of mathematical errors experienced by learners in the foundation phase as a result of the constructivist approach to teaching mathematics should be investigated, and the comprehension and implementation of the new approach by parents should be empirically researched. Lastly, teachers' ability and willingness to provide guidelines to parents should also be empirically researched.

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