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Motivating primary-school learners in mathematics classrooms

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This article explores ways in which primary school mathematics teachers in Mangaung township, Bloemfontein motivate their learners to understand mathematical concepts, to make sense of instructions, and to solve mathematical problems. Data was obtained from class observation, with conclusions being drawn after analysing the discourse. The results indicated that approaching mathematics as a game can motivate learners, resulting in the acquisition of knowledge and skills. The social dimension also plays a role. The use of the mother-tongue (at appropriate times) can be useful in assisting learners to understand mathematical concepts.

Hoe wiskunde-onderwysers primêre skoolkinders kan motiveer

Hierdie artikel ondersoek maniere waarop wiskunde-onderwysers van primêre skole in Mangaung, Bloemfontein leerders kan motiveer om wiskundebegrippe te verstaan, instruksies sinvol te begryp, en probleme op te los in wiskundelesse. Deur die waarneming van onderwysers se klasse is data versamel en nadat die inhoud van die lesse bespreek is, is gevolgtrekkings gemaak. Resultate het aangedui dat, deur wiskunde as 'n spel te benader, leerders gemotiveer word sodat hul vermoëns toeneem en die vak tegelykertyd sonder moeite bemeester word. Die gebruik van die moedertaal op gepaste tye kan 'n besondere bydrae lewer, deurdat dit die leerders help om wiskundige konsepte te verstaan.

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There has been considerable research seeking to understand how learners interact with their teachers (Brombacher 1993; Lee & Loeb 2000); how their cognitive development evolves (Archenhold *et al* 1980; Becker & Selter 1996; De Corte 2000); how they learn mathematics (Watson 1976; Sproule 2000); how motivation affects teaching and learning (Koen 2000; Abdullah & Clements 2001), and about mathematical learning as a social construction of meaning based on the modelling of reality (Brodie 2000; De Corte 2000; Pak, Ismail & Nayan 2000). A number of researchers have recently studied these issues from various perspectives involving the social dimension of cognitive development — the social dimension relating to empowerment (Ernest 2001). In sum, the research reveals that the development of learners is not confined to "any specific aspect" but that it coheres with other aspects (Strauss 2002: 6). The teaching and learning of mathematics is thus much more than a classroom activity and should be perceived as a process transcending the limits of the school.

Koen (2000: 16) describes mathematics as a fascinating game which learners can enjoy. He argues that mathematics teachers and learners need to act in ways that go beyond the physical classrooms and the cognitive aspects of the mathematics being dealt with. In particular, both teachers and learners need to be encouraged to consider the structure of the content, and the context in which learning takes place. These two aspects have an impact on how learners will learn and view mathematics. Setati (1996: 112, 1998: 39) by contrast, is of the view that code-switching (the moving from a foreign language to the mother tongue) can be used to loosen-up the formal procedure encountered in the discourses pertaining to the teaching of mathematics. What seems to evolve from these two viewpoints is that it is always difficult to make learners understand and manipulate mathematical functions, as a result of various antecedents, one of which could be rules. With this in mind, it is logical to attempt to determine whether motivation is an appropriate vehicle for mathematics teachers in primary schools to use in assisting learners to master mathematical concepts and functions.

The objective of this article is thus to determine whether motivation can be used as a vehicle to elucidate meaning and knowledge with regard to the construction of mathematical concepts. It argues further that teachers' actions play a vital role in motivating learners to understand the rules embedded in the game of fractions, and that "code-switching" plays a central role in mediating the understanding of concepts in mathematics classes (cf also Adler 2001).[1]

1. Theoretical framework

1.1 Vygotsky's zone of proximal development

The main theoretical tool used in this article is Len Semenovich Vygotsky's Zone of Proximal Development (ZPD). The underlying assumption of ZPD is that psychological development and instruction ought to be socially ingrained. That is to say, in order to be maximally effective, teachers need to analyse and take account of the surrounding community and its social relations. Due attention needs to be paid to the mental, emotional, spiritual, aesthetic, and social development of learners, since these aspects help to shape the form and content of the instruction offered by teachers.

A good way for children to acquire desirable learning habits is for teachers to model the kinds of behaviour they want their students to emulate. Daniels (1996: 171) quotes Vygotsky as stating that, within certain limits, children are able to copy series of actions that surpass their own capacities. By this means, they are able to perform much better when guided by adults than when left alone, and can do so with increasing understanding and independence. The difference between the level of tasks that can be solved with adult guidance and modelling, and the child's current level of independently solved tasks is the zone of proximal development.

Thus, Sierpinska & Lerman (1996: 848) argued that ZPD is primarily concerned with the difference between what children can do on their own and what they can do with the help of more experienced people. What learners see others do today is likely to be what they will try to do with others tomorrow, and independently thereafter. This means that education is a social activity which calls for skilled teaching personnel to guide inexperienced individuals by applying whatever

1 Recent researchers whose work has contributed to the conception of this article include Ernest 2001, Becker & Selter 1996, Brodie 2000, Ditshego 1999 and Setati 1996 & 1998.

mediation tools (physical as well as symbolic objects) are available. The teacher is often an experienced person who, as a consequence of her superior content knowledge, mediates between the learners and the subject by code-switching — moving from English into the learner's mother tongue, thus enabling the learners to begin to understand the concepts, strategies, and rules embedded in the game.

Vygotsky had an enormous influence on Jean Piaget (cognitive development) and Jerome Bruner (the discovery method), as well as on curriculum development, although he cannot be compared with great mathematicians like George Polya (who argued that the idea is born in the mind of the child and that the teacher should act like a midwife) and Richard Skemp (who analysed the inter-relations between mathematical topics), among others (Watson 1976). His theory transcends the cognitive domain to consider social interaction of learners, and thus forms the basis of this article's argument.

1.2 Sfard's theory of reification

Anna Sfard's (1991) theory of reification also gives substance to the investigations summarised in this article. At the core of this theory is the idea that mathematical notions can be thought of as belonging to two fundamentally different domains. The first is the structural domain, where mathematical thinking is in terms of objects; the second is the operational domain, where mathematical thinking is in terms of processes. Sfard considered these to be complementary rather than separate entities, and maintained that successful learning and problem-solving require the ability to move from one domain to the other. Reification concerns the moment when learners see functions as objects on which they themselves can perform mental operations. This process of individualising mathematical knowledge can be facilitated by conversation — even if the conversation involves only learners, paradoxically, reflecting by "talking to themselves" (Sfard *et al* 1998: 46).

According to Bowie (2000) reification is concerned mostly with becoming familiar with a process and learning to carry it out through mental representations. The element of condensation reflects a gradual quantitative change involving a sequence of mathematical operations that require a quantitative shift in understanding. This shift occurs when the learner is able to break the impression from the process that produced it, and to see it as an object in its own right. Fraction manipulation might be regarded as a set of activities which learners can deal with literally, as objects, or which they can regard as a means of grasping and communicating mathematically with others or with themselves. The mathematics teacher, as an experienced and knowledgeable person, has the capacity to intervene meaningfully and educationally in the mathematical activities of learners. Koen (2000: 11) maintains that in order to generate intrinsic motivation, a teacher needs to handle teaching and learning situations in special ways which take account of both the contexts and the learners involved.

1.3 Motivation

Motivation has to do with the desire or the urge to know or to learn. A person can be motivated to gain knowledge about a phenomenon which is of interest or has a significant meaning in his development and induces a certain pressure from within — what is termed intrinsic motivation. Engelbrecht *et al* (1985: 49) views intrinsic motivation as being

> ... related to the learning situation and [...] determined by such factors as the meaningfulness of education, its purpose, the inner striving of the pupil towards self-activity, self-realization, values, norms, standards and the will to arrive at [...] intellectual maturity by means of education.

The opposite of the intrinsic motivation is externally directed and is known as extrinsic motivation. This type of motivation results from such external factors as favourable circumstances or favourable environmental influences such as an ideal teacher, prizes, the allocation of marks, certificates or rewards (Engelbrecht *et al* 1985: 49). Mokhaba (1993: 73-4) states that motivation should be viewed in the same light as metacognition and emphasises their common ground by citing Harrison:

> Metacognition and motivation ... contain similar constructs that are essential ingredients in the learning process, foremost of which are awareness of: (a) goals/ideal state, (b) current state and resources, (c) perceived distance between current and ideal state, (d) steps needed to reach goals, (e) environmental conditions that affect strategies, (f) knowledge of success.

Motivation plays an important role in any pedagogical situation as it lies at the root of a learner's wanting to learn and persisting in his or her efforts. All learning is in a sense dependent on motivation and teachers need to create an atmosphere that will motivate learners to pursue their goals through education. This is particularly important in the teaching of mathematics as this is regarded as a difficult subject to teach and learn. Hawkey (1995: 47) asserts that because of an emotive reaction — mathematics anxiety, which is induced by one's perceptions of and attitude towards mathematics — "a person may develop psychological blockages which will seriously impair his or her mathematics performance".

Ditshego (1999) used cards to show that mathematics can be treated as a game. In playing the game, teamwork was emphasised, with the winning team being the one that fares best on 24 mathematical manipulations. Setati (1996, 1998) performed seminal research into mathematics teaching in disadvantaged black schools in South Africa demonstrating in particular that code-switching can enable learners to interact mathematically with others. She argued that if the language of a learner is not used regularly in mathematical discourse, then mathematics will remain formal and procedural for that learner. Mathematics has its own language with various forms of communication. The significance of mathematical language is to broaden understanding and it is therefore important that its symbols, which represent powerful ideas and operations, be understood. Code-switching can help learners who have not mastered mathematical discourse to understand the semantics of the relations between symbols and their referents.

2. Methodology and research design

This project is of a descriptive, exploratory kind and employs qualitative methodology. Two grade five classes were randomly chosen from the targeted ill-resourced school in the Mangaung township outside Bloemfontein, and an attempt was made to examine in detail the activities of teachers and learners in the classrooms by means of Textually-oriented Discourse Analysis (TODA). TODA is a three-dimensional model of discourse analysis advocated by Fairclough (1992, 1995), under the influence of "critical linguistics", which focused mainly on the relationship between grammatical structure and the social context in which language is used, and on the implications in terms of power relations. In developing his model Fair-clough borrowed ideas from the work of Michel Foucault on the relationship between discourse and power/knowledge, the discursive construction of social subjects and knowledge, and the role of discourse in social change, because of the contribution of these concepts to social theory (Fairclough 1992). The present study attempted to elucidate the tactics consciously or unconsciously employed by the teachers during mathematics lessons as they attempted to instil motivation in learners and to teach them the rules of the "game" of fractions manipulation.

The method was not based on quantitative procedures and paid little attention to positivist concepts such as external validity, reliability, null and research hypotheses, prediction, universal laws, or causal relations (Mahlomaholo 1998: 202). Rather, the strategy concentrated on identifying and describing the "voices" of the main actors in the game. What the actors said, or tried to say, was used as evidence to authenticate interpretations of data. To ascertain validity and reliability, analytical triangulation of the data was undertaken by comparing and cross-checking the consistency of the information derived at different times during the class visits.

3. Procedure and context

The area and level of the curriculum dealt with was grade five mathematics (intermediate phase — Grades 4 to 6). The intermediate phase is one of the four phases (the other three are preschool, foundation, and senior of the general education and training band which forms part of the eight qualification levels of the National Qualification Framework (NQF) developed by the South African Qualification Authority (SAQA). At the intermediate phase, learners are said to be capable of recognising and understanding relationships between materials, incidents, circumstances and people, and able to deduce the consequences of such relationships (Lemmer & Badenhorst 1997: 164). Mangaung township, where the selected primary school is situated, is one of the poorest black townships in South Africa and its facilities are inadequate, to say the least.

Permission to gather information from the school was granted by the principal. The school caters for about 1000 learners in makeshift corrugated-iron buildings. The principal told the researcher that there were five Grade 5 classes, and agreed to the random selection of two of them (hereafter referred to as Class A and Class B) for the purpose of gathering information. After discussing the aim of the study with the teachers of the two classes, the researcher was permitted to observe mathematics lessons on "fractions" for a period of one week. While observing the classes the researcher took notes, and tape-recorded proceedings in order to capture some of the comments made by teachers and students. The last two lessons of the series were video-recorded.

Class A comprised 51 students, 31 girls and 20 boys, aged from 11 to 14 years. The female teacher was a Setswana-speaking woman in her thirties, with a university diploma in secondary education (UDES). She had seven years' experience of teaching mathematics — four in a secondary school and three in a primary school. Although English was the language of instruction in mathematics (Lemmer & Badenhorst 1997: 70; Masitsa 2004: 214), she often communicated with her learners in Setswana. She could also speak two other African languages. Class B comprised 58 students, 33 girls and 25 boys. The teacher was a male, and had been teaching for less than two years after gaining his senior education diploma (SED). He also spoke Setswana as well as English to his class. Students in both classes were permitted to communicate in Setswana or English. Code-switching was thus used to help learners comprehend difficult concepts and rules. In Vygostky's terms, both teachers attempted to mediate in the zone of proximal development as they presented the content. There was co-operation between the learners and the teachers, with the result that the learners related affectionately with the teachers in their endeavour to develop new knowledge (Brodie 2000: 146). Although the levels of receptive and expressive English of the focus groups cannot be compared to those of their peers who speak English as a first language, they were able to move from the structural domain to the operational domain of mathematical thinking, as defined by Sfard's theory of reification (Sfard 1991).

4. How the game of fractions manipulation is played

Various mathematical activities were conducted during the course of the week. Since space does not permit discussion and analysis of extended quotations, discourse and brief excerpts have been chosen to illustrate how the game of fractions was played in the classes.

4.1 Lesson 1, Class A

In the initial phase of the lesson Teacher X asked the learners questions about the work previously done. She then wrote the heading "Fractions" on the chalkboard, as it was the subject to be learnt in the 45-minute lesson, and asked the learners to take out a clean sheet of paper so as to start with the game. The following is the transcript of what ensued:

> Teacher X: Fold it nicely. It must be equal. ["E lekane" meaning it must be equal]. What comes in your head? [Oh! come on]. What is a fraction? [Ao!, le lebetse?, Itekeng. Have you forgotten? Try]. What comes into your head?

At this point the learners seemed to be unsettled, possibly because the researcher was a stranger in the classroom. Noticing this, the teacher used Setswana to calm them down. Koen (2001: 11) states that where time is limited in the classroom the teacher should correct mistakes by the quickest possible method. She repeated the question in their mother-tongue and changed the strategy for getting the answer, using English:

> Teacher X: Can any one of you go and write a fraction on the board? (At this point most of the learners ran to the board and she called them back, letting one girl write). What fraction has she written on the board?
>
> Learners (answering in chorus): One-third.
>
> Teacher X: Good [Le tota le le botlhale. You are really clever].

As learners were actively involved in solving problems in fractions, the teacher realised that some of them seemed to be confused about the concepts "denominator" and "numerator". She asked those who knew the difference to help their classmates to work it out. The active participation of learners in helping others was accompanied by motivational expressions from the teacher such as "come on", "try" and "you are very clever" demonstrating a caring attitude, as espoused by constructivists). The method seemed to lead to free engagement in intellectual mathematical problem-solving (Hawkey 1995: 190), although it was difficult to distinguish between those learners who really understood what was being presented to them as the class was very big and that not every learner understood the mathematical language, despite being in the same classroom. The learners' enthusiasm about working as a group exemplifies Vygotsky's ZPD from the vantage point of both acquisition and participation metaphors. A teacher entrusted with assisting a learner has to listen and guide where possible. Mokhaba (1993: 161) asserts that a teacher's responsibility as far as explaining to learners is concerned is to clarify why certain phrases or notations are ineffective. At the end of the game of fractions the teacher said:

> Teacher X: The boys won the first activity by being able to paste the right name of the fraction on the given space. They have one point. The girls won the second activity by recognising the most errors. So, we have a draw. (At this point Teacher X divided the chalkboard into two, showing the scores of each group).

The learners were pleased that the game of fractions had resulted in a draw. Before dismissing Class A, the teacher gave the students some homework.

The manner in which information is presented can have an impact on the interaction among learners and with the teacher. If the learners do not understand the concepts and cannot conceptualise what the teacher is trying to impart to them, another form of stimulus should be brought into the classroom. Learning involves dual activities, that is, thinking and active mental engagement, and does not exclude an interactive social process that is elucidated by the use of language and sign systems (Brodie 2000: 133). As the mental and the social are complementary rather than separate entities, motivation during teaching activities is crucial if learners are to be able to move from the structural to the operational domain as posited by Sfard's theory of reification (Sfard 1991).

4.2 Lesson Two, Class B

This lesson followed the same pattern as that with Class A, except that colours were used in an attempt to emphasise salient features of fractions:

> Teacher Y: Today we are going to do two sets of fractions — coloured fractions and non-coloured fractions. On the left-hand side I have non-coloured fractions and on the right-hand side the coloured fractions. Do you follow? [Aah! la tlhaloganya? — Do you follow?].
>
> Learners (answering in chorus): Yes, Sir!
>
> Teacher Y: On the table in front of you are little paper designs of fractions. So, I want you to give them names according to the way they are coloured. I want you to say the fraction with its colour. [Dirang ka pele — Be fast. Basimane ba kae? — Where are the boys?].

The teacher prompted learners to write on the chalkboard and to demonstrate to others how to solve the problems they encountered. This approach emphasised hands-on activities such as comparing paper strips, lengths, colours and patterns. Mathematical concepts were used so as to assist learners in acquiring mathematical language that they would use in reflection. The internalisation of mathematical language and learning makes it easy for learners to reflect on their own thinking (Becker 2001: 6). Switching between the language of the learners and English (since the Department of Education's policy is that English should be used as the medium of instruction in schools) makes it possible for learners to comprehend the rules used in the teaching of fractions. Individual effort is also needed from learners because "learning is not merely a question of apprehending mathematical truths but a question of entering a community of practice" (Abdullah & Clements 2000: 249). The game continued in the same manner until all the fractions had been covered. Teacher Y's concluding words before the end of the lesson were:

> Teacher Y: So, in this case nobody wins. The score is one each. It is a draw.

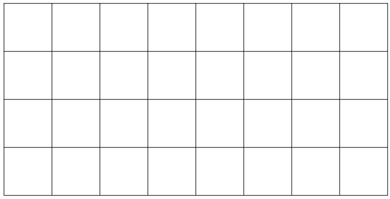
Learners were then reminded to write down their homework before leaving. The teacher moved around to help whenever possible and "consistently posed word problems in each group's zone of proximal development" (Murray *et al* 1992: 2). The encouragement that the teacher brings to the classroom assists learners to grasp and understand the complexities involved in the manipulation of concepts.

5. Observations and findings

Mutual co-operation prevailed between the teacher and the learners as well as among the learners, who gave one another a chance to participate in the game of fractions. A learner would suggest to a group how to approach a problem, the suggestion would be considered, a reason would be given why a certain strategy should be chosen, and an agreement would be reached. This atmosphere of horizontal partnership became more prevalent when colour was introduced into the game. The colouring was not done merely for the sake of making the diagram appear attractive. Both reasoning and the aesthetic component of the exercise seemed to be at the heart of the activity. The co-operation among the learners and between the learners themselves was an indication that there was a "didactic contract" (Mokhaba 1993: 160) among all the parties, a vital component of effective mathematics teaching.

Colour game

Colour a $\frac{1}{4}$ of each block in two different ways.



Learners would compete by placing different coloured shapes in a block of their own choice to form a coloured pattern. The group that finished first with many patterns would be declared the winner.

There is no single best way to teach mathematics. Rather, an integrated and responsive approach, taking into account the background and needs of individual learners, is the best option. This article has problematised and evaluated the types of communications used in classrooms during mathematics lessons. It has tried to shed light on the role played by the mother tongue in the learners' understanding of certain "essential" mathematical rules and concepts. It has also drawn attention to the role that teachers can play as mediators of learning — a role which makes them vitally important agents for motivating learners to want to understand the rules, principles, and values of the game that school calls "mathematics" (Koen 2000: 13).

Although the classrooms studied were overcrowded, the teachers tried very hard to create order and establish interest, by grouping the learners. This was evident when the teachers asked who could write a fraction on the board or who could help a group that had got a sum wrong: almost all the learners ran to the front. Nevertheless, the teachers did not lose sight of their objectives or of the needs of individual learners. In fact, they called on students by name and sometimes evoked panic by saying that those caught making any noise would be required to remain behind during play-time.

Although the students participated as groups in which leaders had been appointed, the teachers were inclined to use direct mathematics instruction in guiding learners to solve any problems they encountered or in introducing new concepts. Group leaders directed and intervened whenever a group went astray, but members were afforded enough freedom to develop answers and strategies of their own.

The way in which the teachers involved the learners in the subject matter was highly stimulating for the learners, who enjoyed themselves through all the phases of the game of fractions. The teachers succeeded in motivating learners by introducing probing questions into every activity — like asking the learners what fraction would remain if they were to cut 2/3 of a chocolate bar in half.

The child-centred approach was skilfully used to assist learners to comprehend the necessary mathematical concepts such as "denominator", "numerator" and the values of fractions. More formal teaching approaches, leaning towards the instructional mode or allowing the learner to explore the content on his or her own, were sometimes used in accordance with the perceived needs of the learners at any particular moment.

The discourse used in all the classes was characterised by everyday language, which kept the lessons fluent as well as enjoyable and accessible for the students.

6. Discussion

The main thesis is that, in order to enable learners to understand and master mathematics, teachers need to motivate them. This can be done by creating a classroom environment in which all learners become actively involved in the processes of clarifying and developing the concepts and skills that they need to learn. The female teacher in Class A did so when she wanted learners to grasp a concept ("I don't see sixteen portions here. Try to concentrate and get the right answer"). This procedure of motivation is consistent with Vygotsky's ZPD, which emphasises the idea that children's interaction with adults can enhance their linguistic and cognitive development since they will tend to model their own language constructions on those of the teacher (Sierpinska 1994: 67). In the presence of skilful and empathetic teachers, learners can, for example, learn to construct and explain the rules relating to fractions (Mudaly & de Villiers 2000: 22). The processes of construction are fragile, however, so the mediation tools (language, gestures, charts and any relevant objects in the classroom) can be crucially important in helping learners learn to play the game.

The teachers in this article were able to switch, in a natural way, from Setswana to English, or *vice versa*, in order to express or clarify fractional concepts ("Fa o di tshwaya kafa le kafa o tla araba dipotsao jaang?" — If you mark them on both sides how will you give the answer?). Setati (1996, 1998) argued that code-switching is necessary in mathematics lessons because without it mathematical discourse will almost certainly remain formal and procedural. Sierpinska (1994: 67) pointed out that the language of the teacher is not only based on mathematical symbols and terminology, but also employs everyday spoken language and didactic jargon. Each of these aspects of language has its own conventions, and one aspect might not be compatible with another. To overcome this in teaching, various rules and pro-active measures can be used to maintain the motivation and the interest of learners.

The learners were empowered to overcome the problems they encountered because the teachers were able to move from one domain to another. When the male teacher noticed that his Class B learners were not able to identify the fractions on the chart, he wrote them on the board. Because learners were not conversant with using graphs for fraction-solving, the teacher had to adjust. This is consistent with Sfard's (1991) notion of reification, in terms of which successful learning and problem-solving depend on a learner's ability to move from one domain to another.

When a learner was distracted by activities outside the lesson, the teacher helped him/her to re-focus by posing direct questions. This was evident when the male teacher said to one girl: "Mosetsana bowa kwa o leng teng" — Girl, come back from where you are. This successful redirection of the learner suggests that the quality of a lesson depends on co-operation between learners and teachers. It is also apparent that learners can be motivated and assisted by having an experienced person guide them back onto the right teaching/learning track.

Teacher X indicated to the writer during a discussion that her learners often did not seem to remember what they had been taught the previous day. She blamed this on the lack of resources at the school — a common factor at impoverished schools — and on the "knowledge explosion", or the way in which processes outside the school environment affect whether interaction between teachers and pupils makes sense to the latter. Cosin *et al* (1977: 45) maintains that the cultural milieu within classrooms represents the combined backgrounds of pupils and teachers, and that what transpires within classrooms is thus as much determined by social forces operating outside the classrooms and the schools as by those within the classrooms.

7. Conclusion

The findings of this article have practical implications for teachers and for their learners' motivation. Teaching mathematics in schools situated in disadvantaged areas is clearly a challenging task which demands patience, dedication and empathy if teachers are to have any chance of real success. In particular, a teacher's need to deal effectively with mixed groups of children is likely to pose a daunting challenge because of the uniqueness of each individual learner. In order to be effective, mathematics teachers do not only need to be conversant with their subject. They must also be flexible in using an integrated approach and code-switching at appropriate times. In these ways, coupled with the use of a reflective-inquiry paradigm in the classroom, mathematics teachers are likely to be able to motivate learners to follow the rules of the game and to enjoy applying them. When learners believe they are being encouraged to know, interact with, and help their classmates during a lesson, they engage more in learning than when the teacher lacks any motivational spirit. Ryan & Patrick (2001: 438) are of the opinion that the classroom environment has a major impact on learners' motivation and engagement since classrooms are inherently social places in which learners learn in the presence of their peers. Finally, it is only through a collaborative effort on the part of the Department of Education, universities, technikons and the corporate sector that such teacher development can benefit learners in studying mathematics.

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